Passive vs Active Attacks

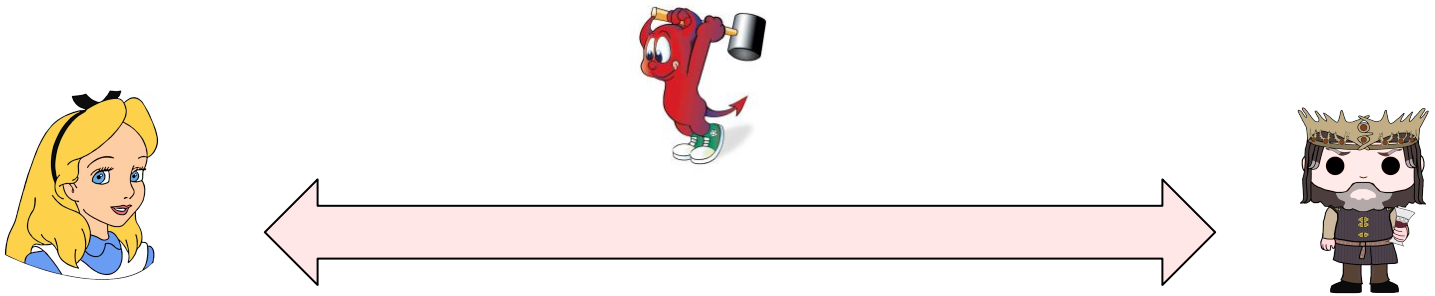
So far we have mainly considered passive attacks

- The attacker simply observed the ciphertexts transmitted over the communication channel
- At best, it influences Alice and Bob's choice of the plaintexts , but it never tampers with the data in transit

Passive vs Active Attacks

We now consider **active** attacks:

- The attacker has full control over the channel



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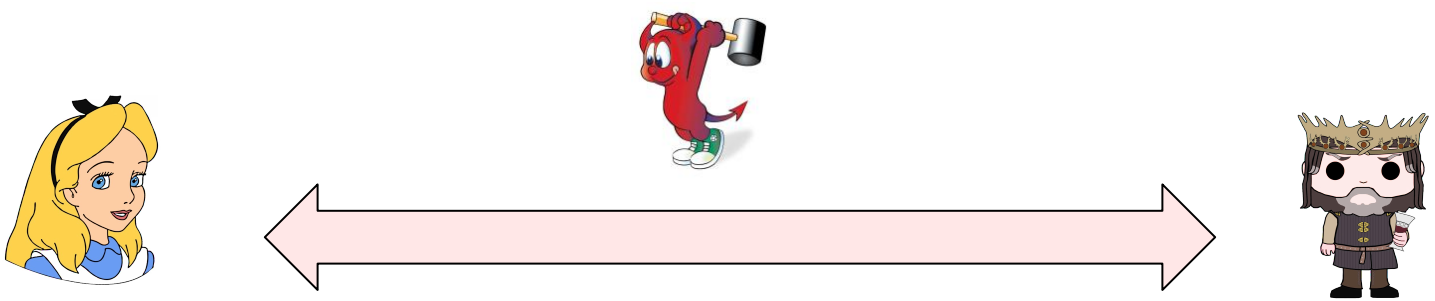

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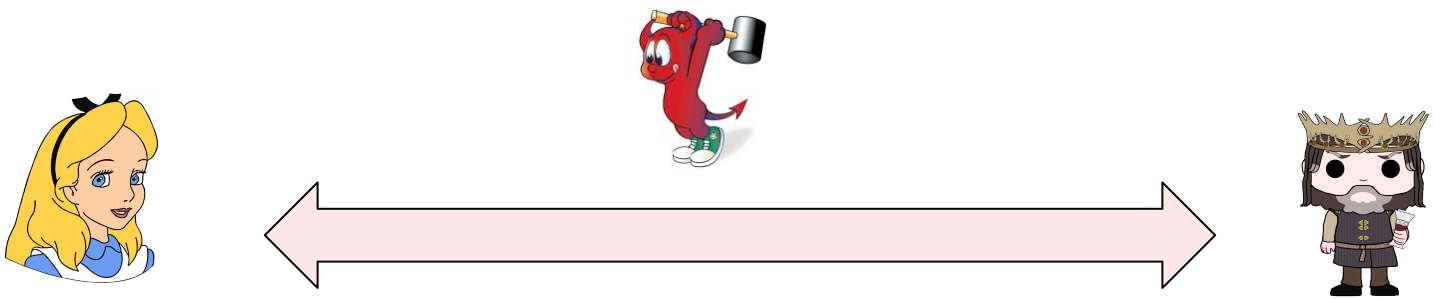

Passive vs Active Attacks

We now consider **active** attacks:

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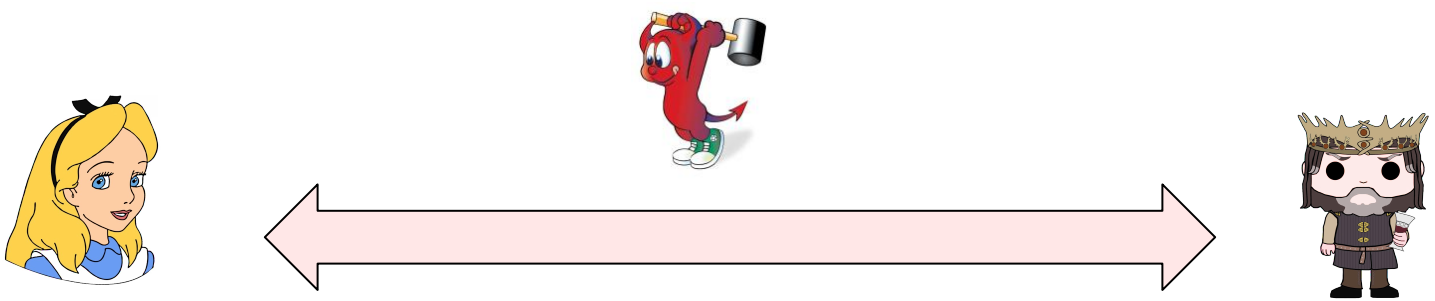
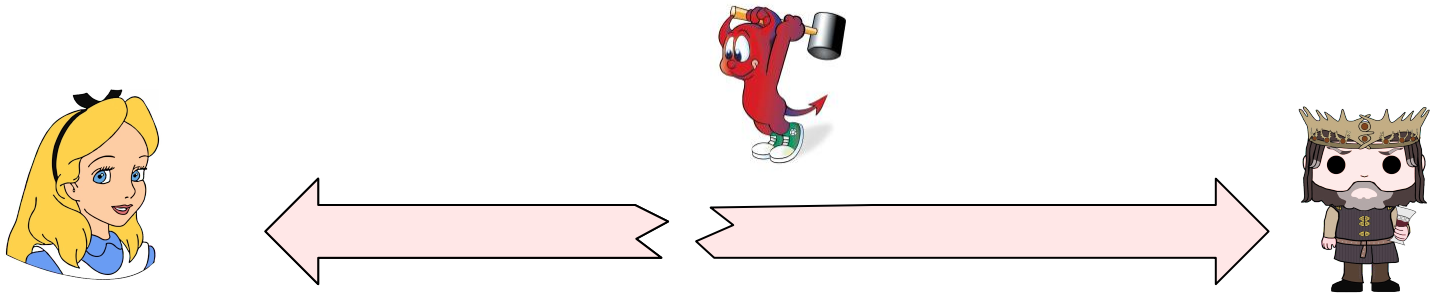
- Can alter the message contents
- Can drop messages
- Can forge new messages



Denial of Service

An adversary this powerful can always stop any communication between Alice and Bob (by simply dropping all messages)...



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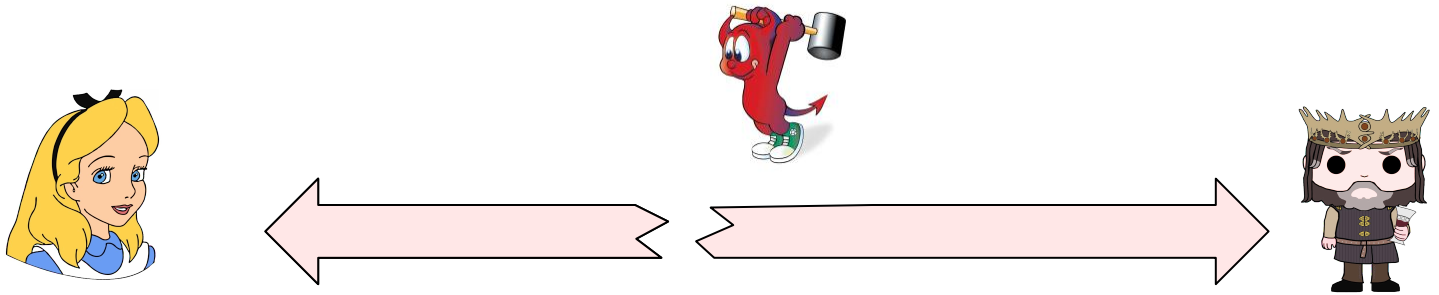

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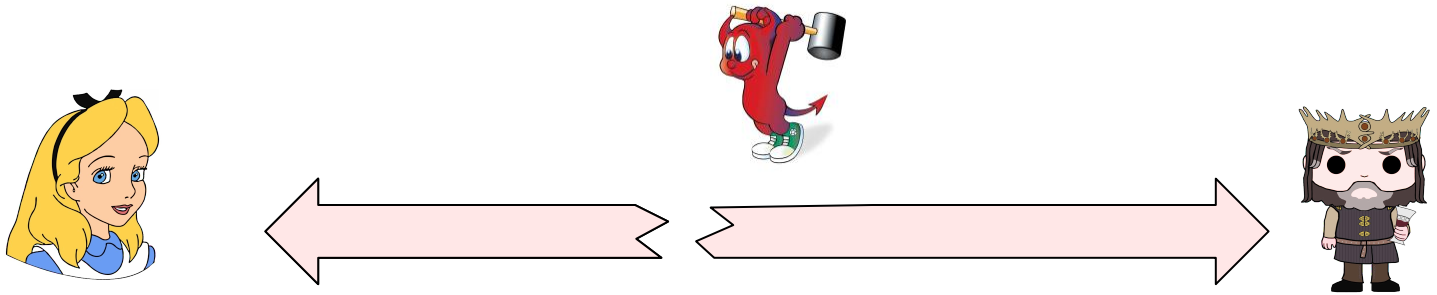

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We are interested in what security guarantees we can achieve *when communication does happen*

Secrecy vs Integrity

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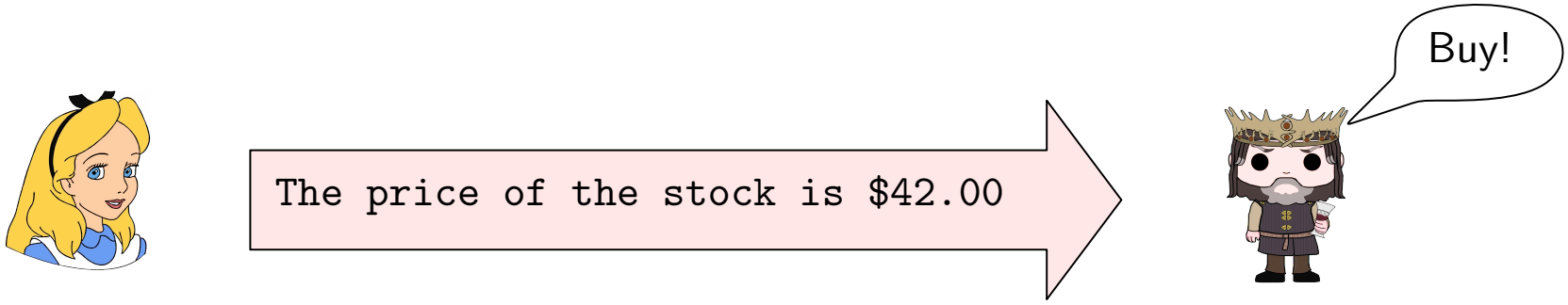
Integrity and Secrecy are **orthogonal** concerns

Secrecy vs Integrity



- Not a secret information!
- No need to encrypt
- Need to check that it comes from a trusted party
- Need to check that the amount has not been tampered with

Encryption schemes for Integrity?

In all the schemes we have seen so far:

- A modified ciphertext can be decrypted without any issue (and it yields a different plaintext)
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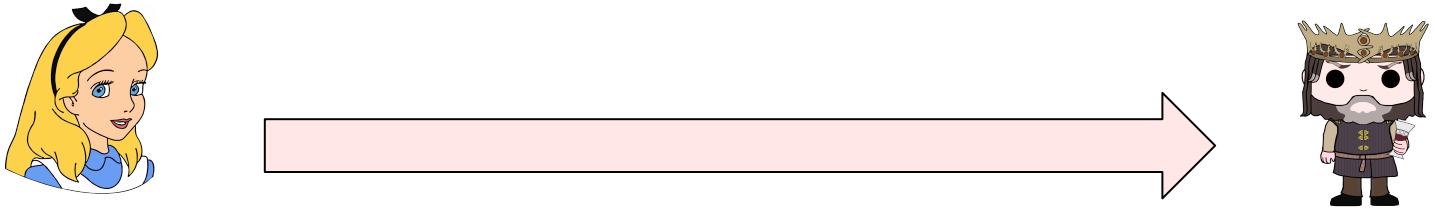
Encryption schemes are not the right tool to guarantee integrity

Message Authentication Codes (MACs)

The right tool are **Message Authentication Codes**



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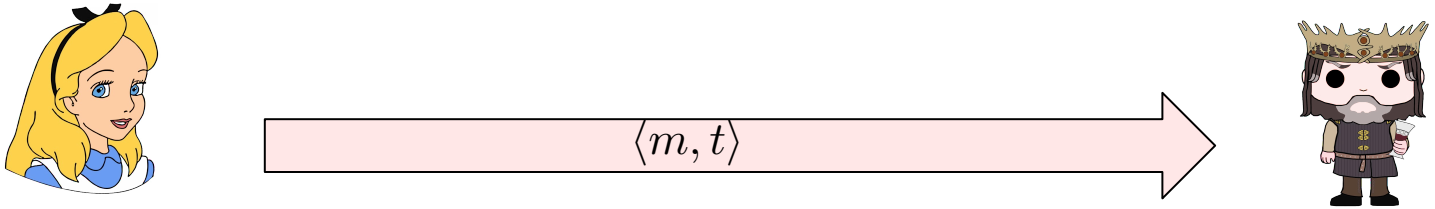

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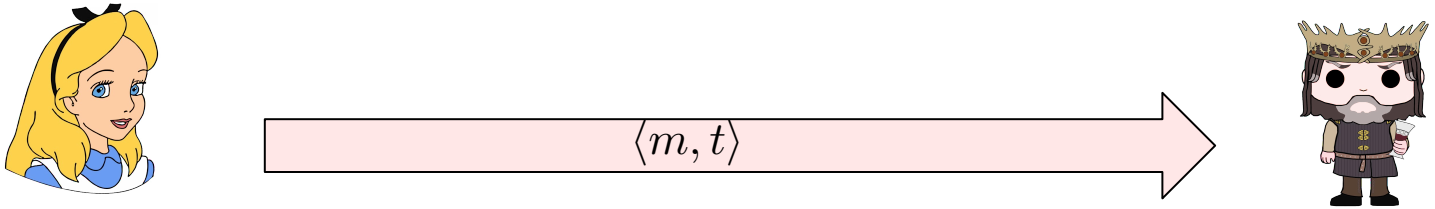

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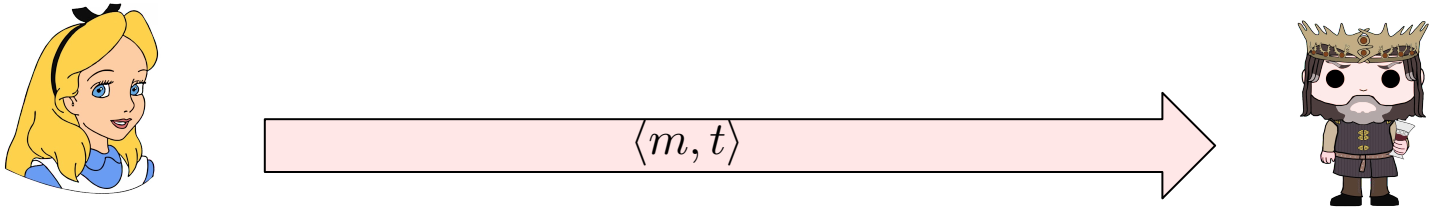

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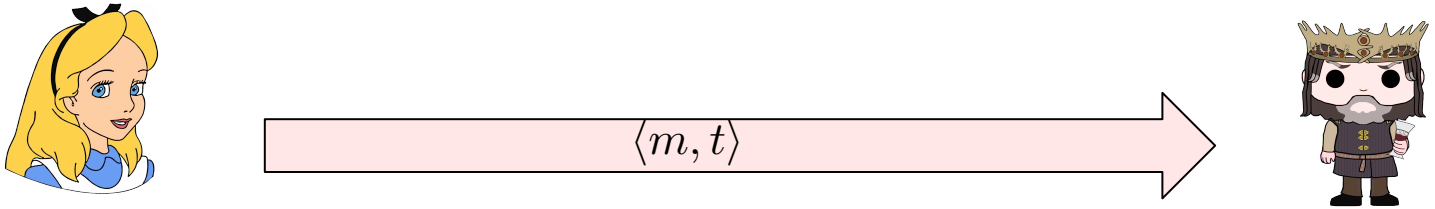

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- Intuitively, no (efficient) adversary can forge t

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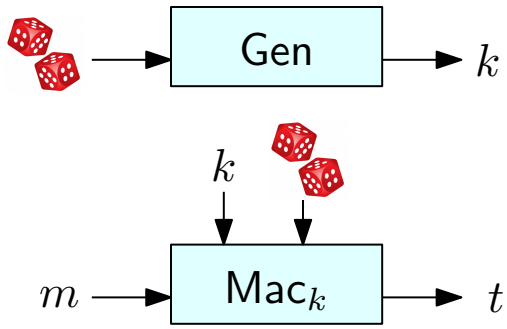

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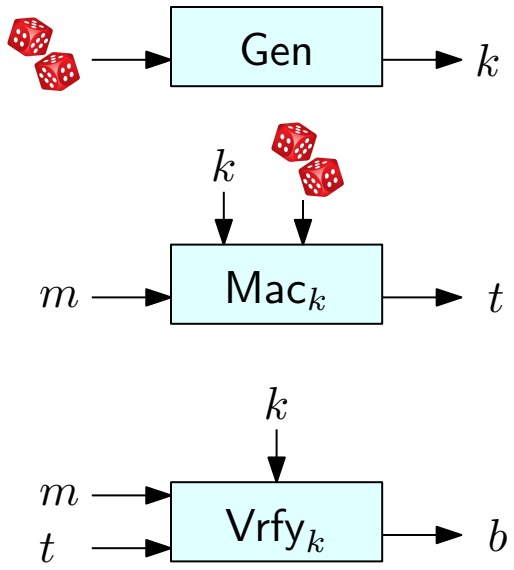

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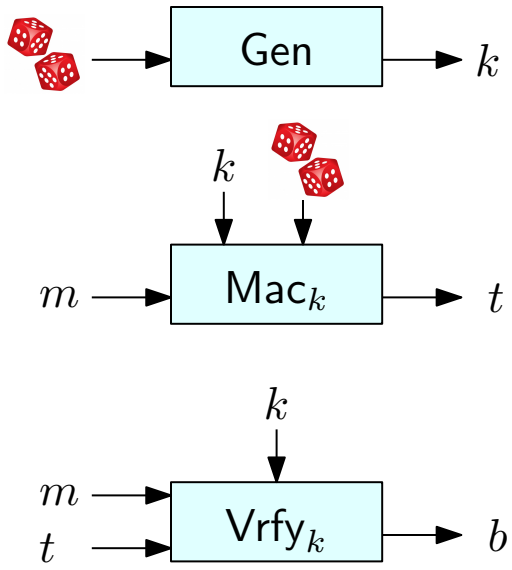

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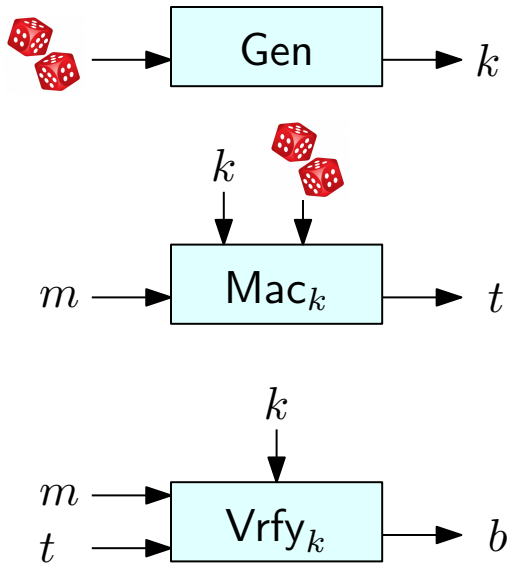

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If Mac is only defined for messages $m \in \{0,1\}^{\ell(n)}$ we call $(\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ a **fixed-length** MAC for messages of length $\ell(n)$.

Message Authentication Codes (MACs)

In the special case in which Mac is a deterministic algorithm, we can use the following **canonical verification** algorithm:

$\mathsf{Vrfy}_k(m, t)$:

- $\widetilde{t} \leftarrow \mathsf{Mac}_k(m)$
- If $\widetilde{t} = t$:
    - Return $b = 1$
- Else:
    - Return $b = 0$

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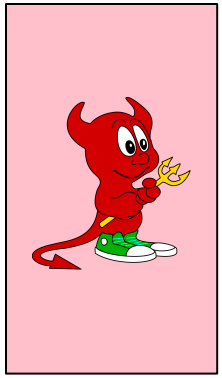
Security Goal: Existential unforgeability

- No efficient attacker should be able to provide a valid tag for any message that was not previously authenticated by the sender, except with negligible probability.

The Message Authentication Experiment

Let $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ be a MAC. We name the following experiment $\mathsf{Mac\text{-}forge}_{\mathcal{A},\Pi}(n)$:

- A key k is generated using $\mathsf{Gen}(1^n)$



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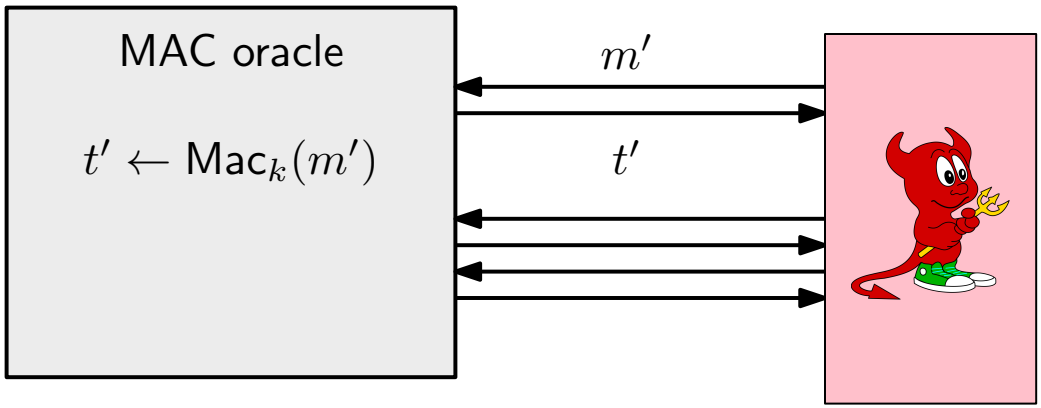

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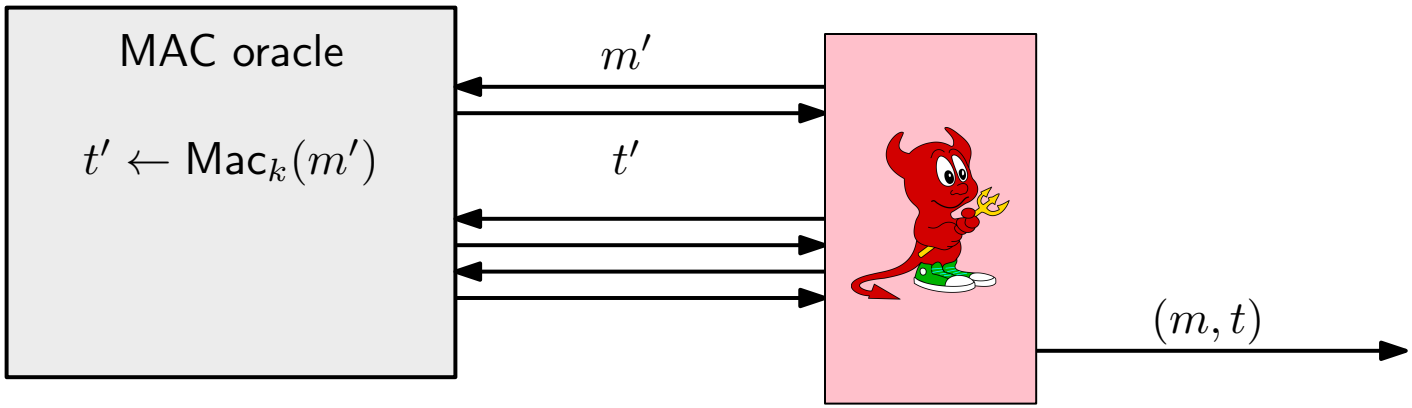

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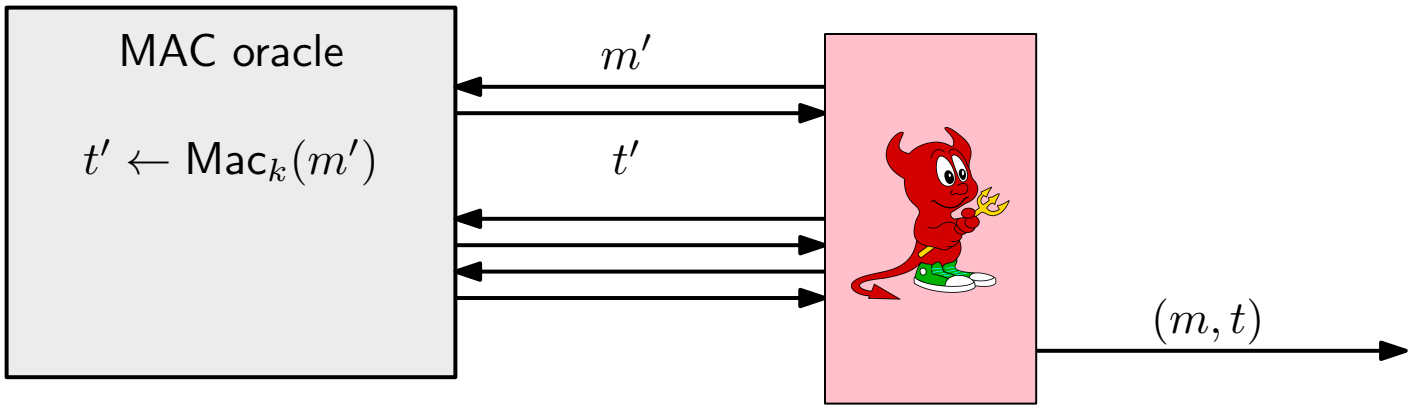
- The outcome of the experiment is 1 if (*) holds and $\mathsf{Vrfy}_k(m, t) = 1$. Otherwise the outcome is 0.

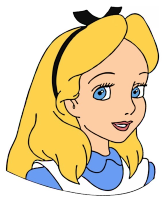


Secure MACs

Definition: *A message authentication code* Π *is existentially unforgeable under an adaptive chosen-message attack (is* **secure***) if, for every probabilistic polynomial-time adversary* $\mathcal{A}$*, there is a negligible function* ε *such that:*

$$\Pr[\textit{Mac-forge}_{\mathcal{A},\Pi}(n) = 1] \leq \varepsilon(n)$$

Replay attacks



Alice's Bank

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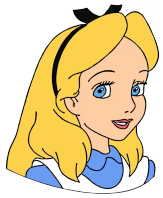

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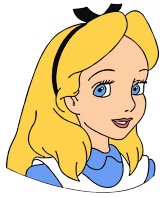

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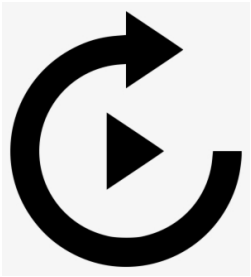

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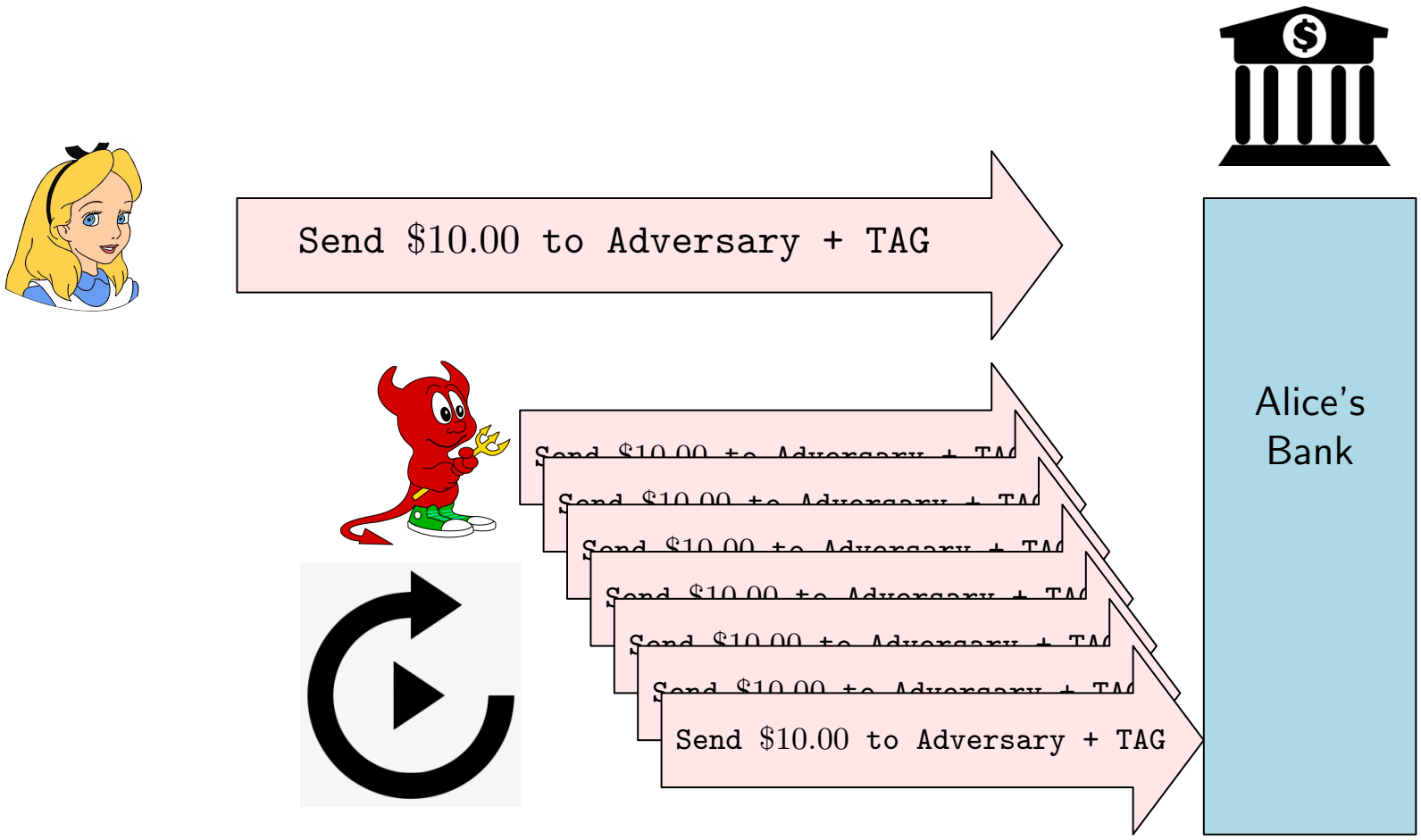

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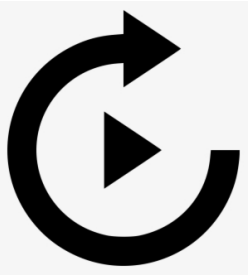

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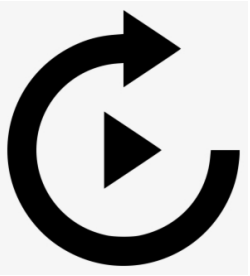

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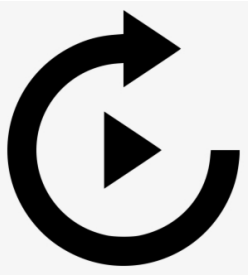

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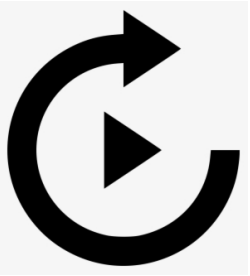

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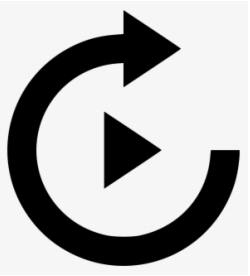

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  Drawbacks:

  - Clocks need to be available and synchronized (this is not trivial in embedded systems).
  - Replays can still happen in a short window of time.

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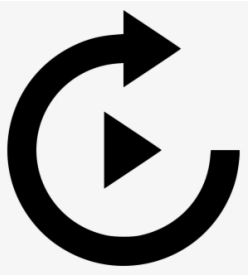

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- How to handle replayed/repeated messages depends on the application

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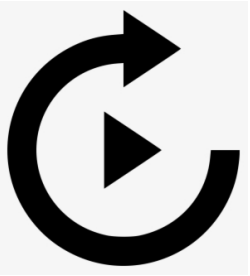
- The sender keeps a counter C, sends the current value of C along with the message, and increments C. The recipient checks that the counters of the received messages are increasing.

Replay attacks

Our security definition does not prevent replay attacks

- No stateless mechanism can prevent them!



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  - Need to keep track of the counter
  - Needs to handle messages delivered out of order

A fixed-length MAC

Intuition: We want some keyed function $\mathsf{Mac}_k(\cdot)$ such that, even if we know $m_1, m_2, \ldots$, and $\mathsf{Mac}_k(m_1), \mathsf{Mac}_k(m_2), \ldots$ it is infeasible to predict $\mathsf{Mac}_k(m)$ for some $m \notin \{m_1, m_2, \ldots\}$

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- $\mathsf{Gen}(1^n)$ returns a random key for F
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Theorem: *If F is a pseudorandom function, then the MAC Π (for messages of length n) constructed from F as above is secure.*

Proof of security

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Usual proof strategy:

- Assume that there is some polynomial-time adversary $\mathcal{A}$ that wins Mac-forge$_{\mathcal{A},\Pi}(n)$ with non-negligible probability
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Reminder:

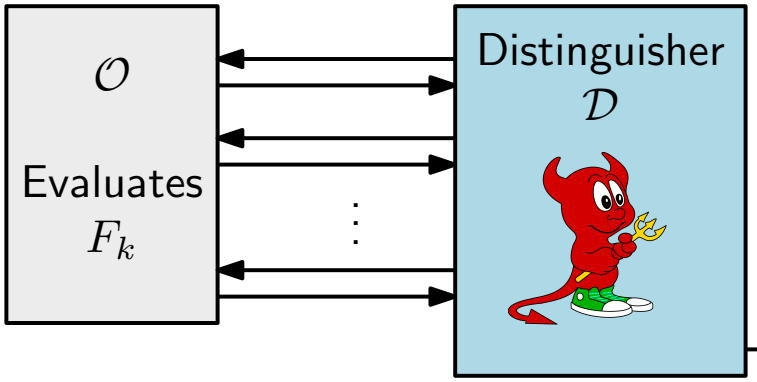
Definition: An efficient, length preserving, keyed function $F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ is a **pseudorandom function** if for all probabilistic polynomial-time distinguishers D, there is a negligible function ε such that:

$$\left| \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \right| \leq \varepsilon(n)$$

Reminder: distinguishers for pseudorandom functions

"World 0":

k is chosen u.a.r. in $\{0,1\}^n$



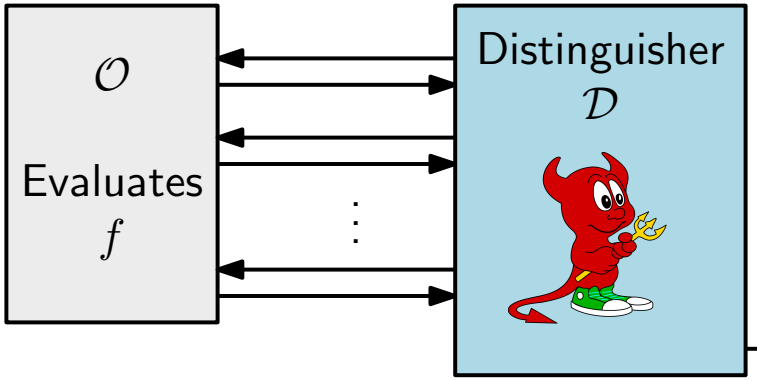
D wants to tell "World 0" apart from "World 1"

Output $(0$ or $1)$

$D^{F_k(\cdot)}(1^n)$

"World 1":

f is chosen u.a.r. in Func_n



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Denotes the kind of oracle D is interacting with

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  - Answer t' to $\mathcal{A}$ (say that t' is a tag for m')
- Whenever A outputs (m, t) (at the end of its execution):
  - Query Φ with m and obtain a response t^*
  - Return 1 iff $t^* = t$ (return 0 otherwise)

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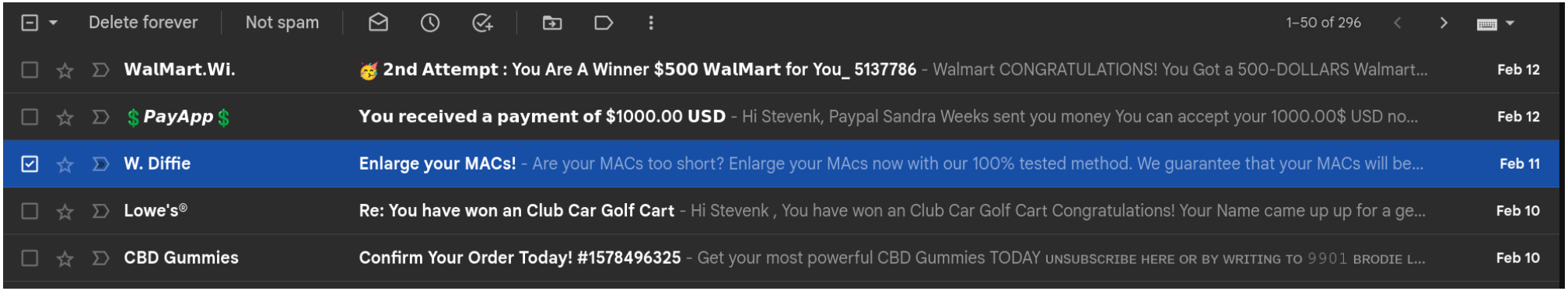
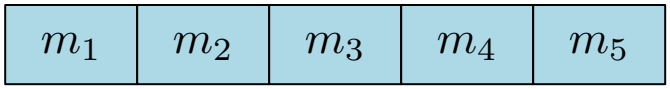
Domain extension for MACs



Domain Extension for MACS

A first idea:

If the block-length is ℓ, we split the message into blocks $m_1, m_2 \ldots$ of length ℓ

$m =$ 

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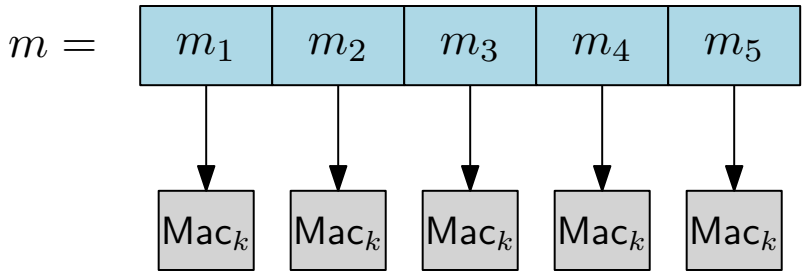

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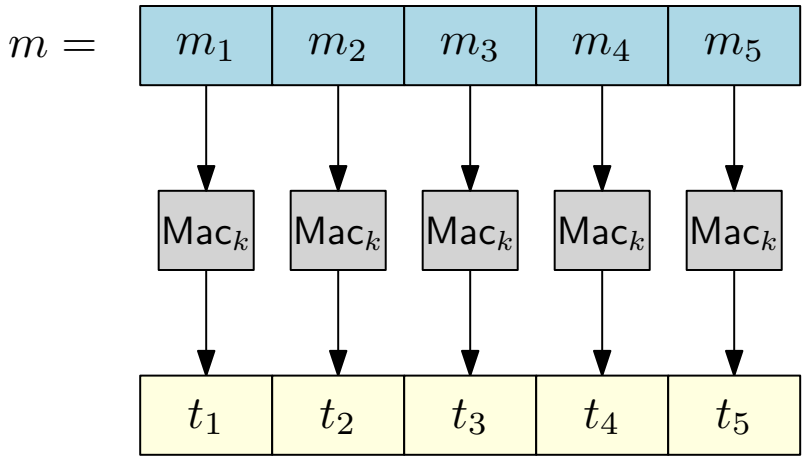

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Output $t_1 \,\|\, t_2 \| \, t_3 \,\| \ldots$

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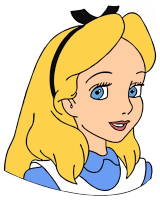
Does it work?



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`Do not`	`attack.`	`Commence`	`retreat.`	t_1	t_2	t_3	t_4

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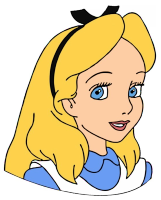

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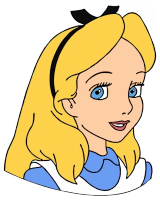

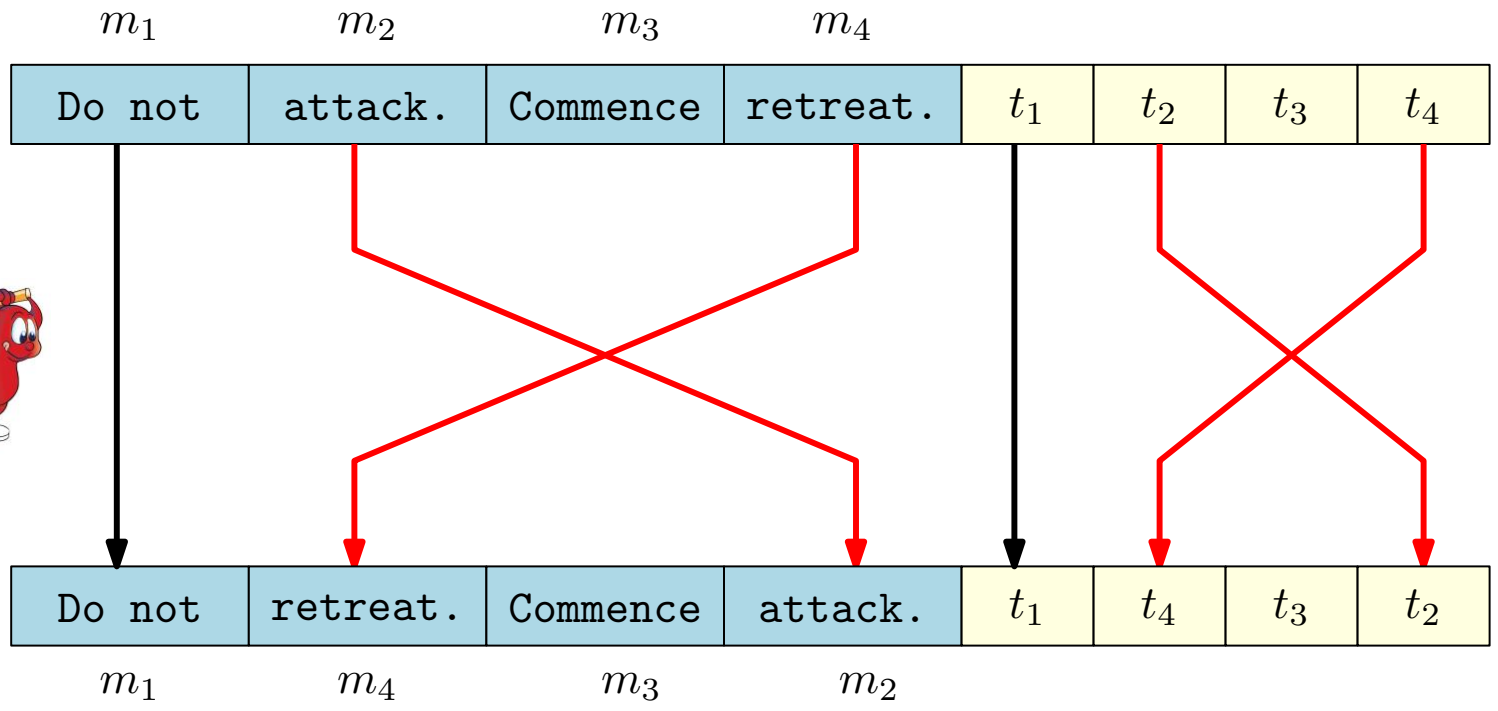

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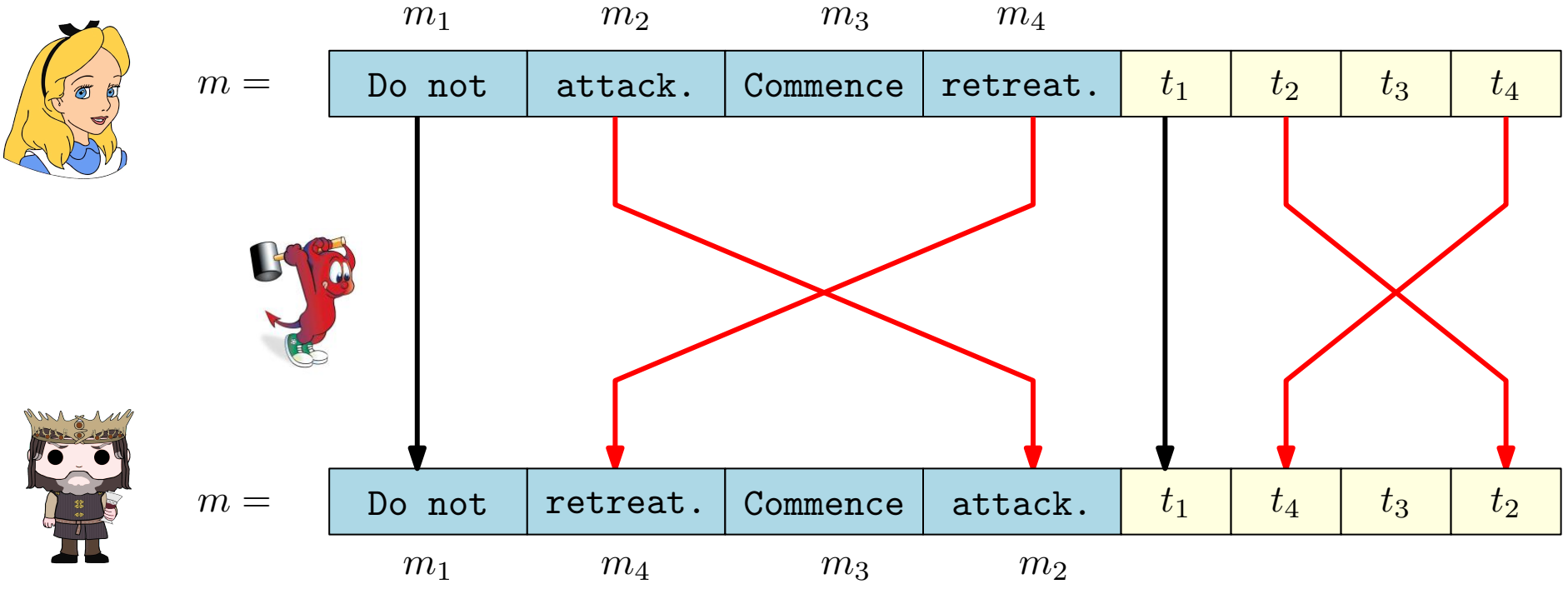

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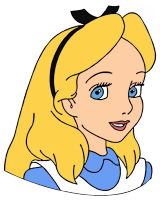
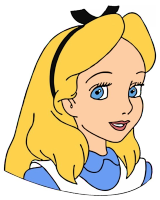
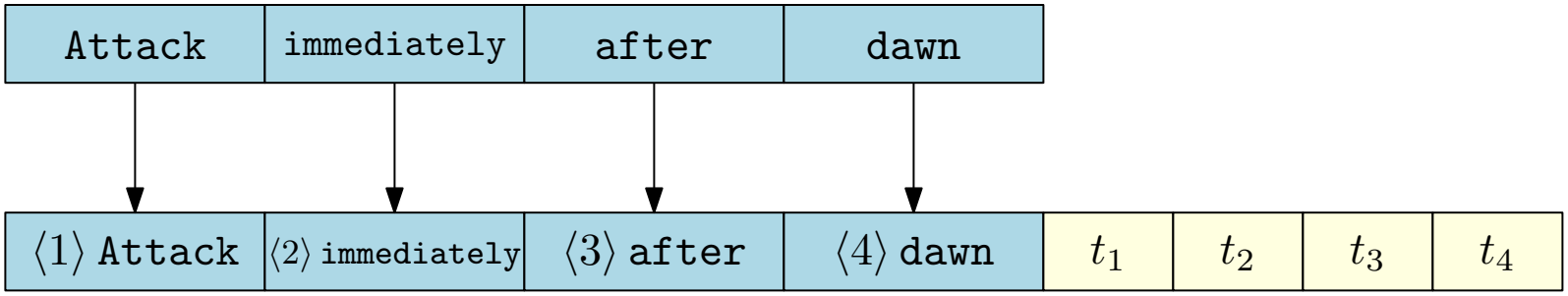


- Vulnerable to **block re-ordering attacks**
- We can prevent such attacks by adding a block index to each block

Domain Extension for MACS

Is the resulting MAC secure?





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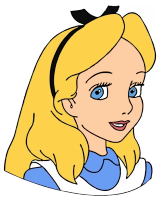

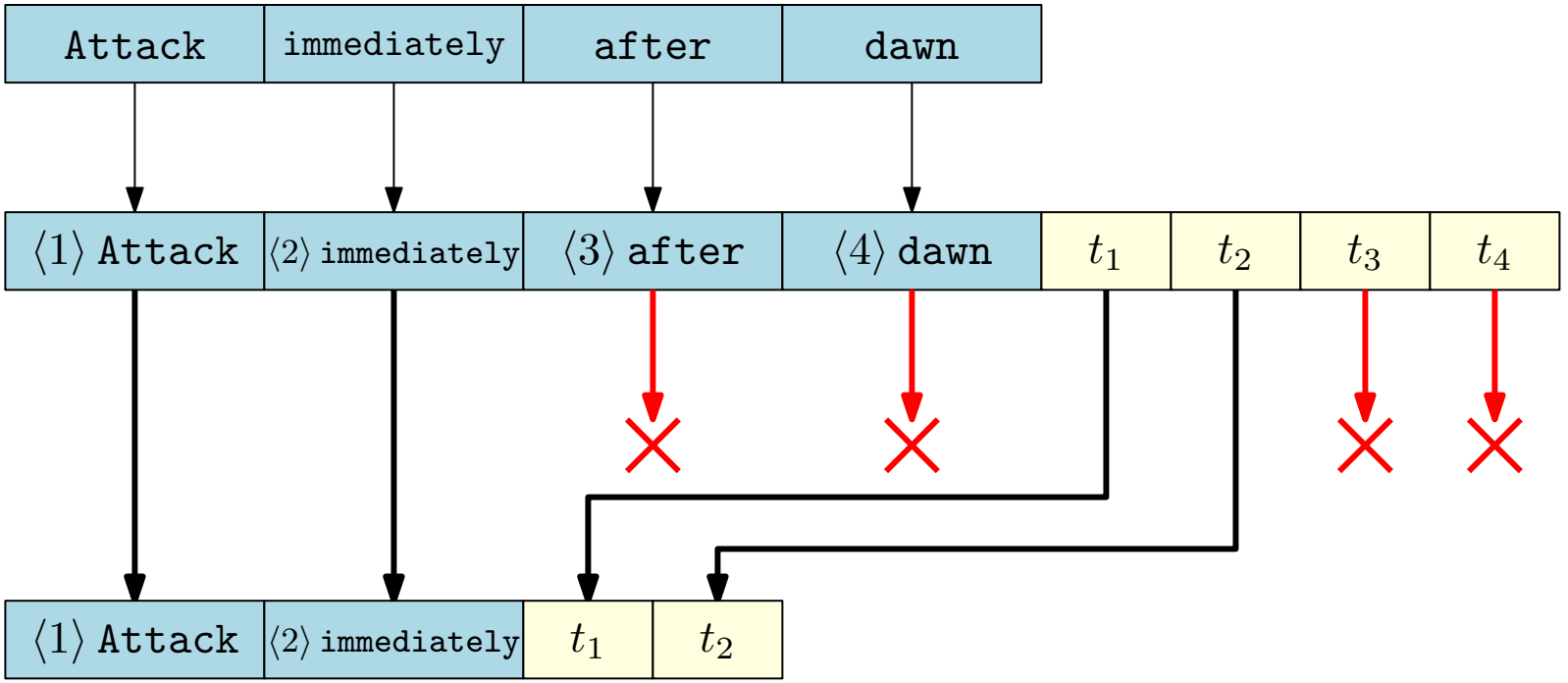

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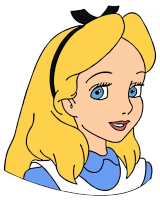

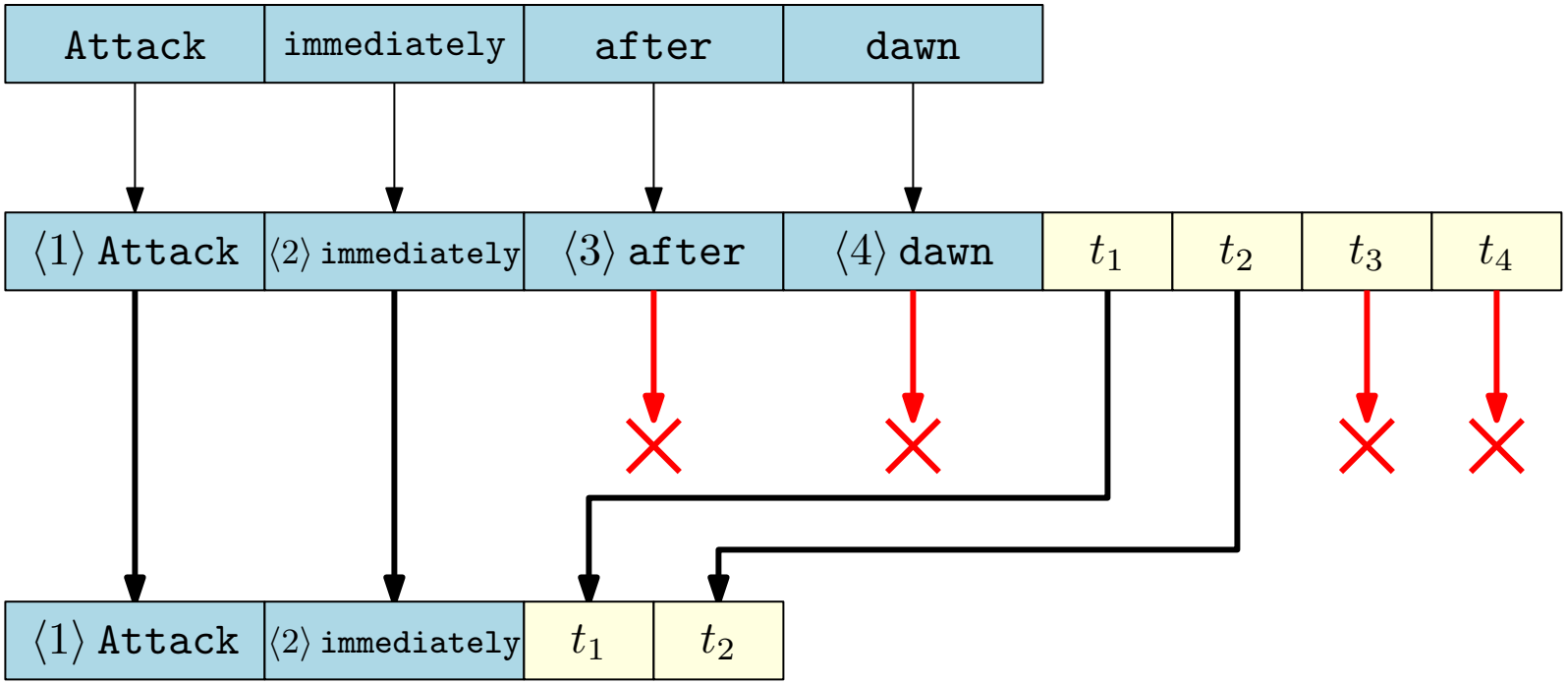

- Vulnerable to **truncation attacks**

Domain Extension for MACS

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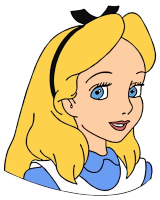

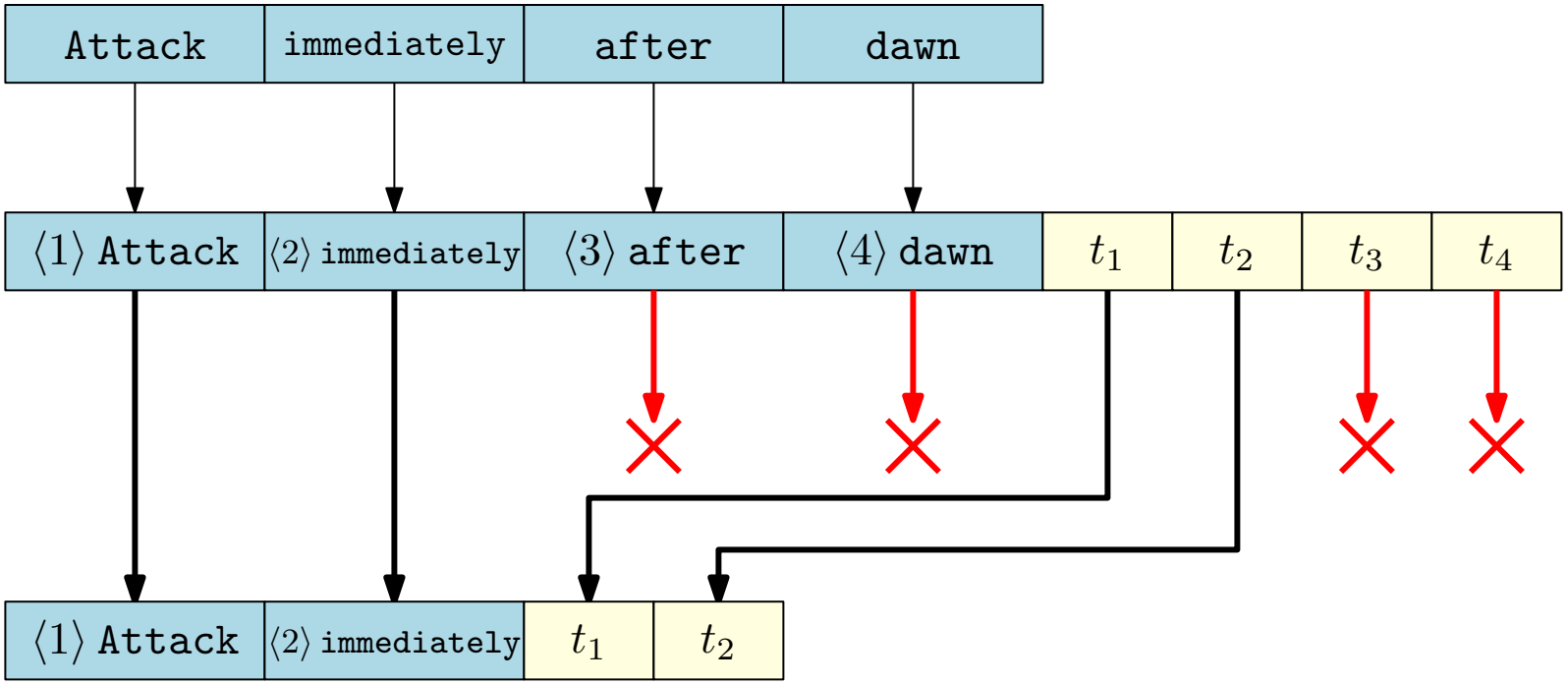

- Vulnerable to **truncation attacks**
- We can prevent such attacks by adding the message length to each block

Domain Extension for MACS

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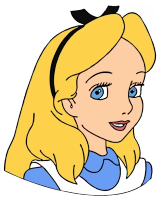

$\langle 27,1\rangle$ `Fire`	$\langle 27,2\rangle$ `John Doe`	$\langle 27,3\rangle$ `for his`	$\langle 27,4\rangle$ `theft`	t_1^1	t_2^1	t_3^1	t_4^1

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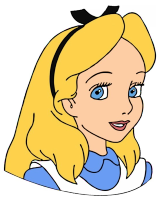

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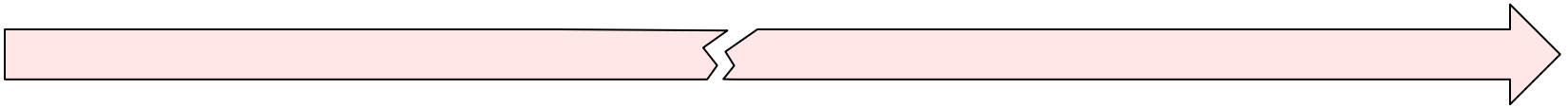

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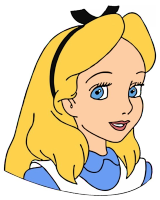

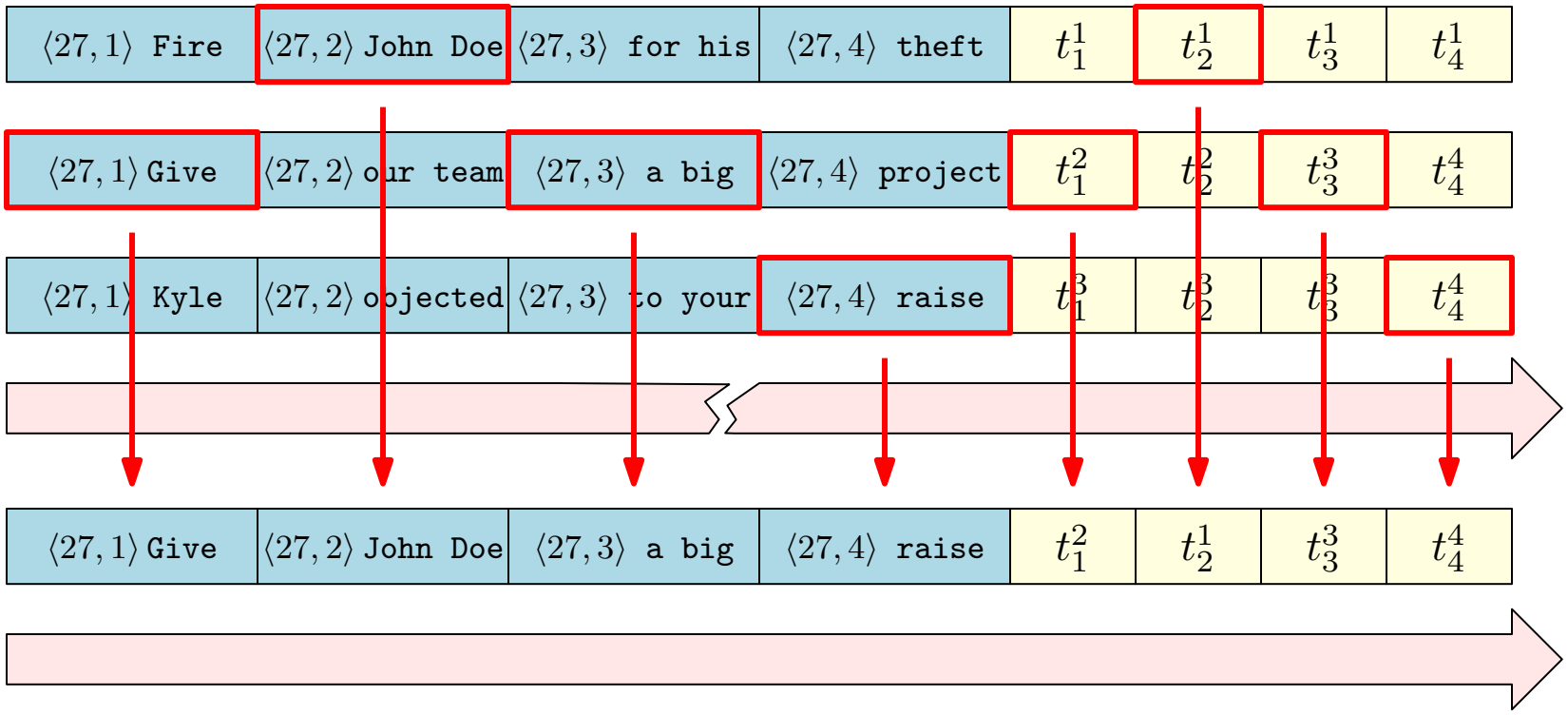

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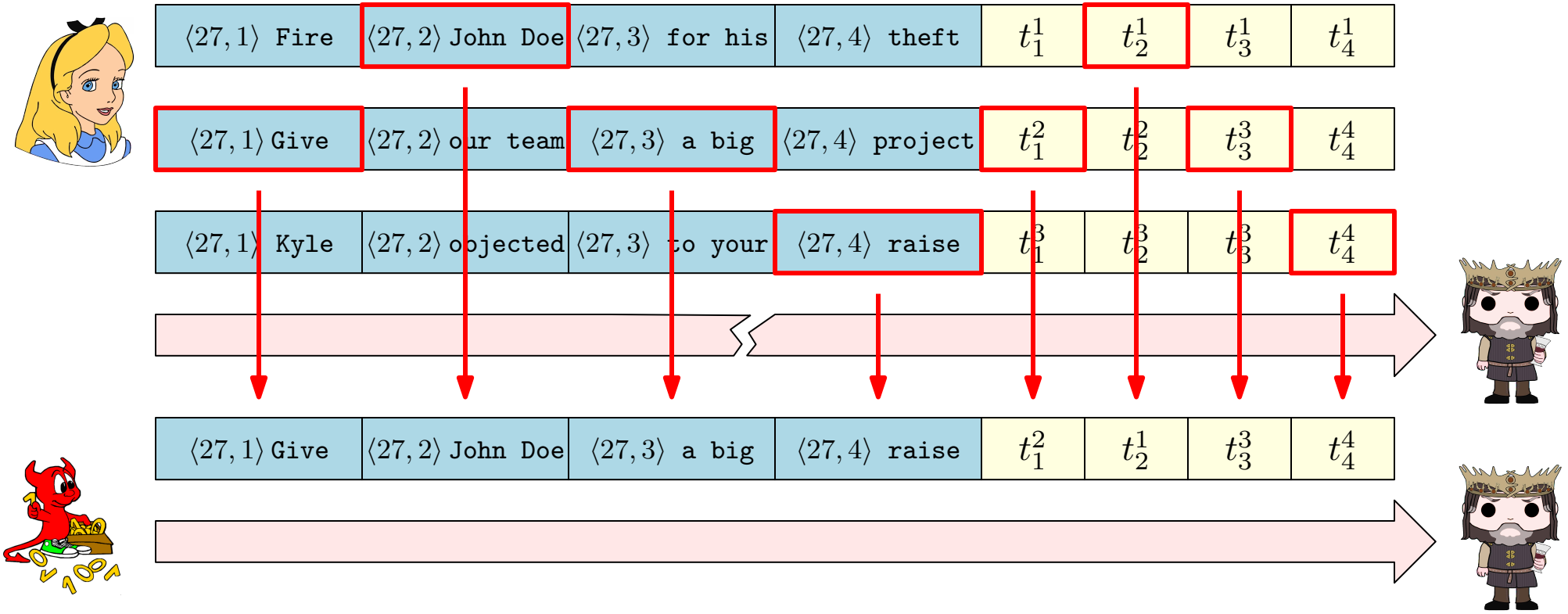

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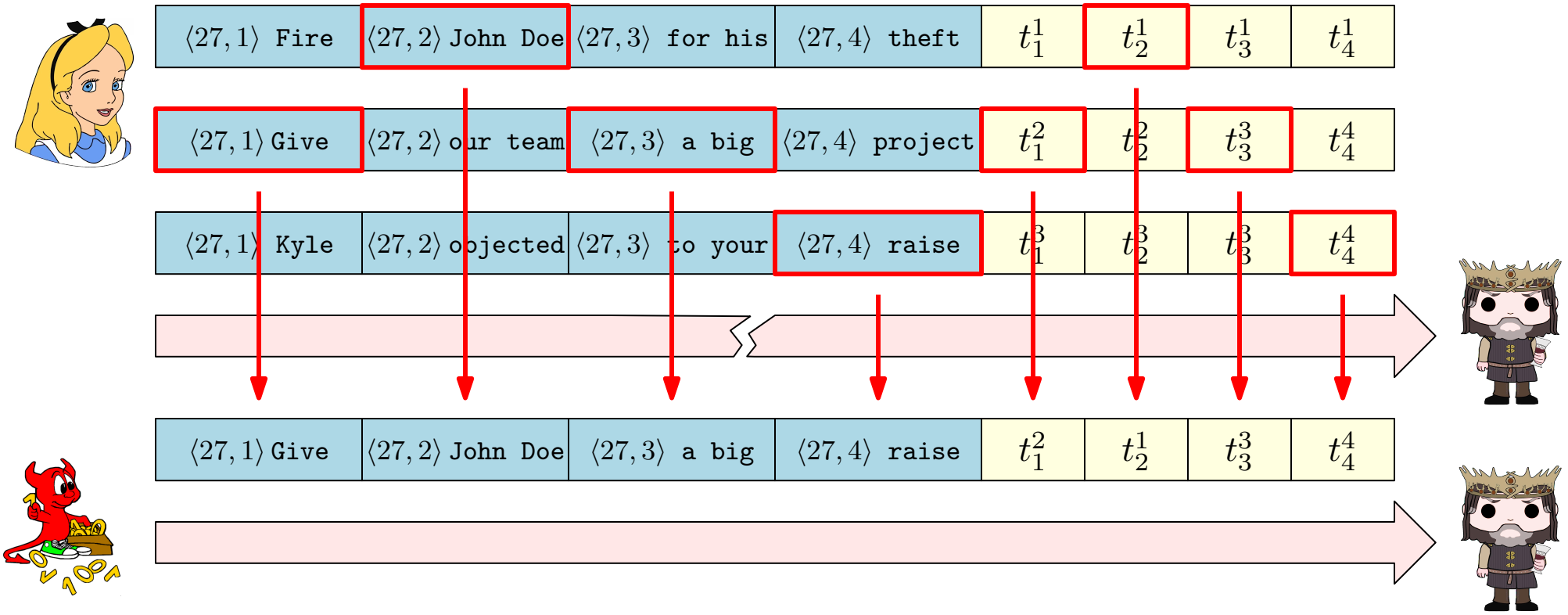

- Vulnerable to **mix-and-match attacks**
- We can prevent such attacks by choosing a random **message ID** and adding it to each block

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We can use $\ell/4$ bits for for each of the message ID, message length, block index, and for the actual payload.

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- Choose r uniformly at random from $\{0,1\}^{\ell/4}$
- Split m into blocks $m_1, m_2, m_3, \ldots, m_d$ of $\ell/4$ bits each (pad the final block, if needed)
- For each $i = 1, 2, \ldots, d$
  - $t_i \leftarrow \mathsf{Mac}_k(\langle r\rangle \,\|\, \langle |m|\rangle \,\|\, \langle i\rangle \,\|\, m_i)$
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Domain Extension for MACS

Theorem: if Π is a secure fixed-length MAC for messages of length ℓ, then Π' is a secure MAC for arbitrary-length messages.

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- To compute the tag for a message of length $|m|$, we need $\approx \frac{4|m|}{\ell}$ evaluations of the block cipher
- The computed tag is long (i.e., longer than $4|m|$ bits)

(Basic) CBC-MAC for fixed length messages

We can do better by using a construction similar to the ciphertext block chaining (CBC) mode used for block ciphers.

The construction only works for messages of **fixed** length $n \cdot p(n)$, where n is the block length of F_k

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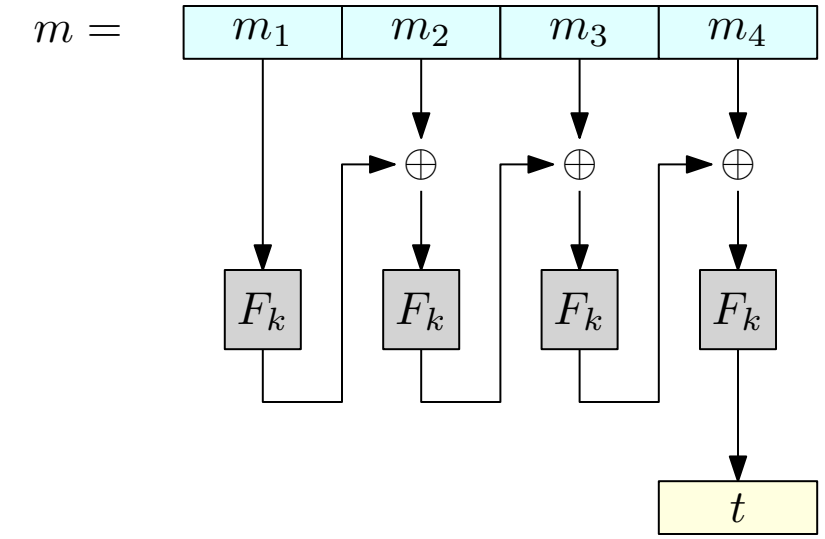
$\mathbf{Mac}_k(m)$:

- Parse m as $\ell(n)$ blocks $m_1, m_2, m_3, \ldots, m_{\ell(n)}$ of n bits each
- $t_0 \leftarrow 0^n$
- For $i = 1, \ldots, \ell(n)$
  - $t_i \leftarrow F_k(t_{i-1} \oplus m_i)$
- Output the tag $t = t_{\ell(n)}$



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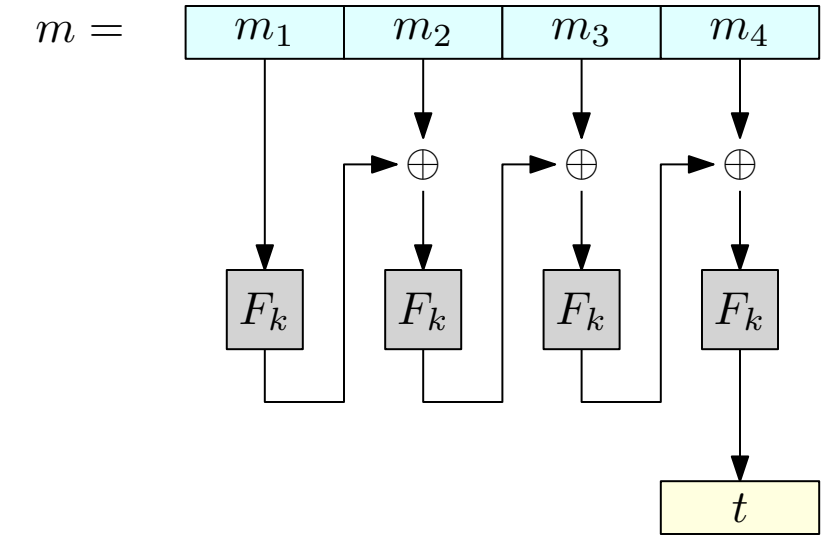

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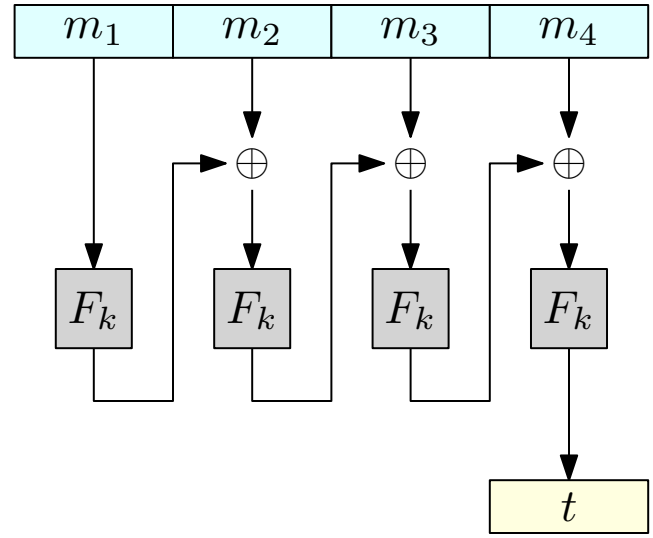

Some differences with CBC mode for block ciphers:

- No IV (notice that CBC-MAC is **deterministic**)
- Only the output of the final invocation of the block cipher is returned

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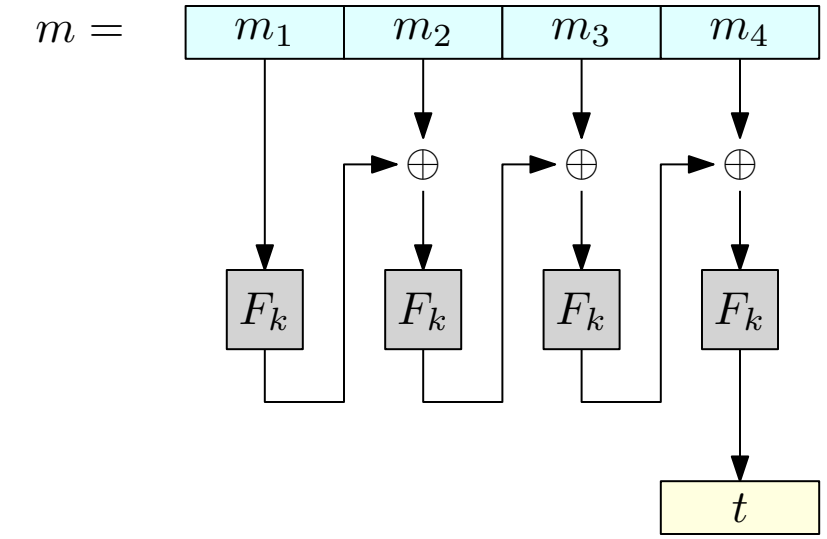
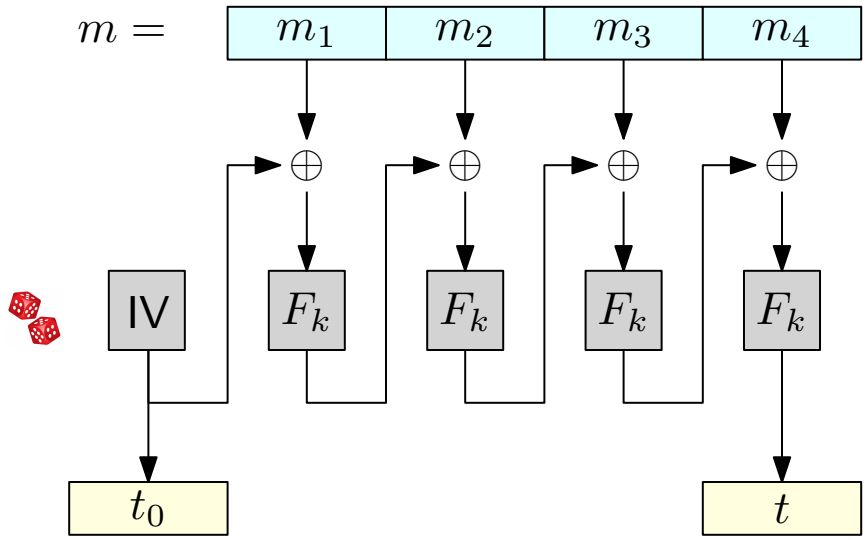


Theorem: Let p be a polynomial. If F is a pseudorandom function with block length n, then Basic CBC-MAC is a secure MAC for messages of length $n \cdot p(n)$.

Basic CBC-MAC: some caveats (1/3)

If we modify the construction to take an IV, then the MAC is no longer secure!



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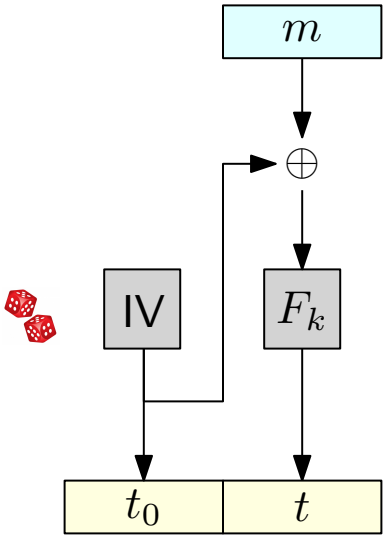

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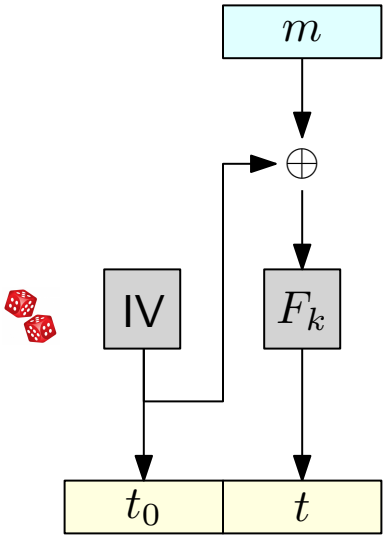
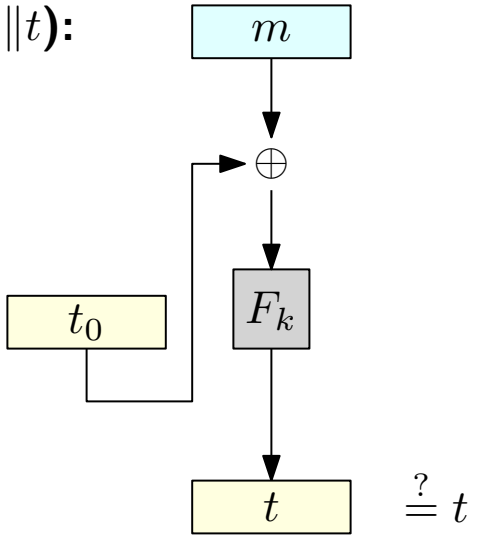


Vrfy($m, t_0 \| t$):



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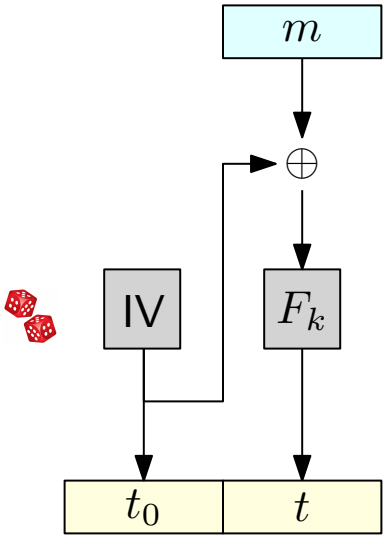

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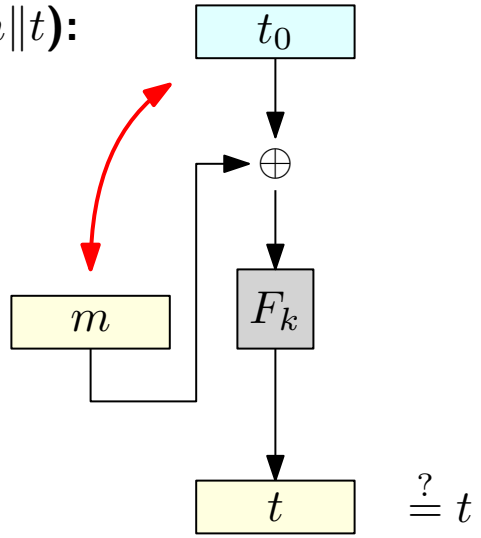

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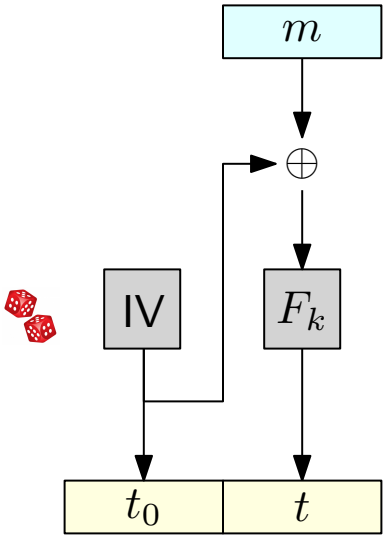

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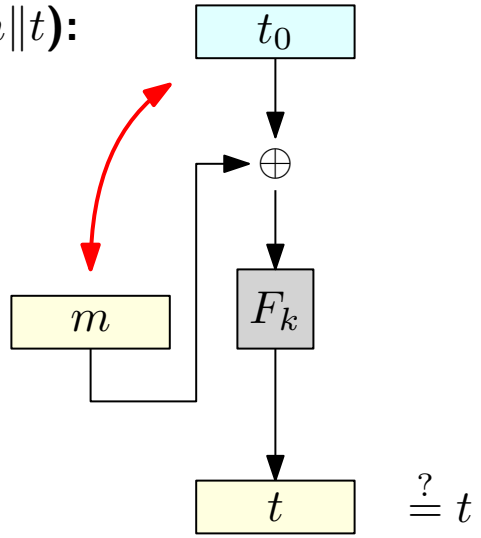

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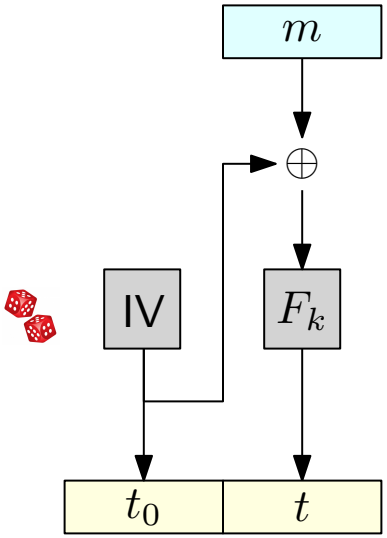

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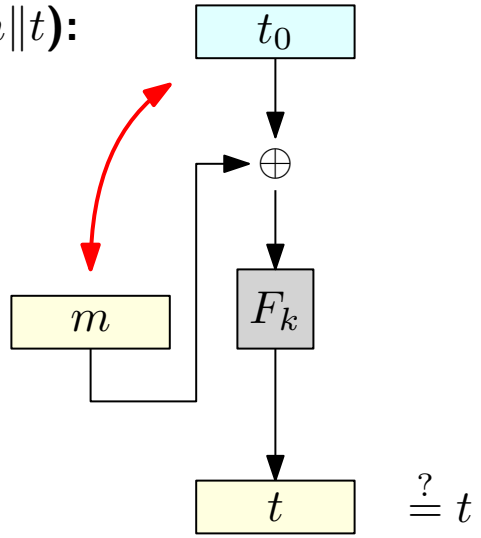

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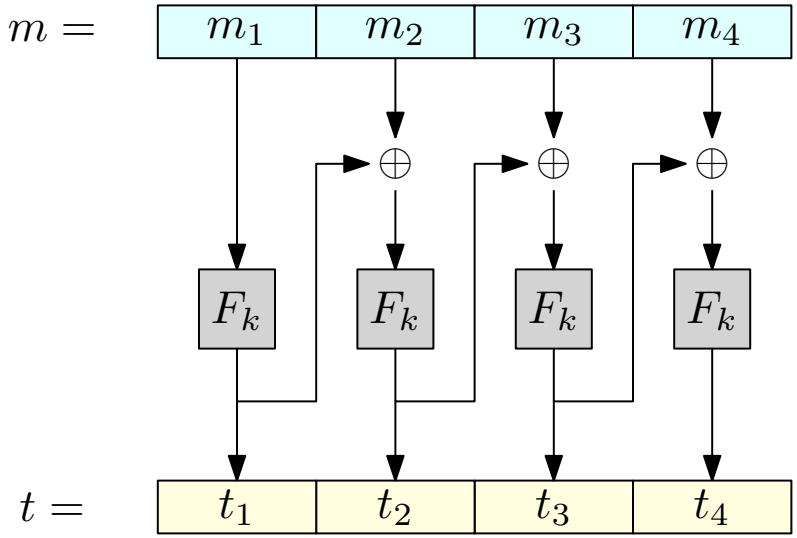
The forgery is successful

Basic CBC-MAC: some caveats (2/3)

If all invocations of F contribute to the output, then the MAC is no longer secure!



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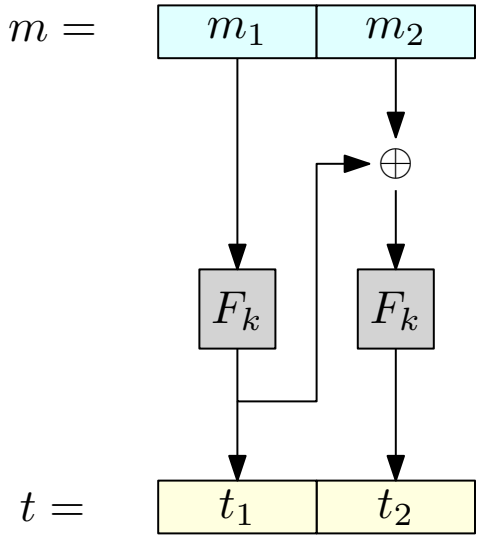

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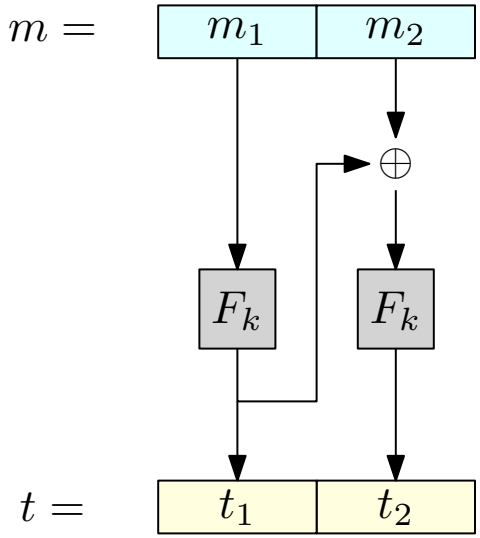

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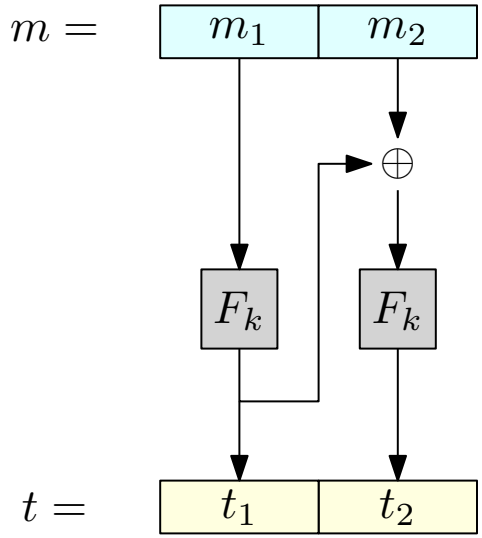

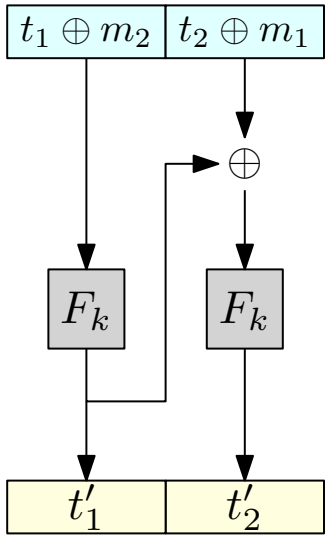

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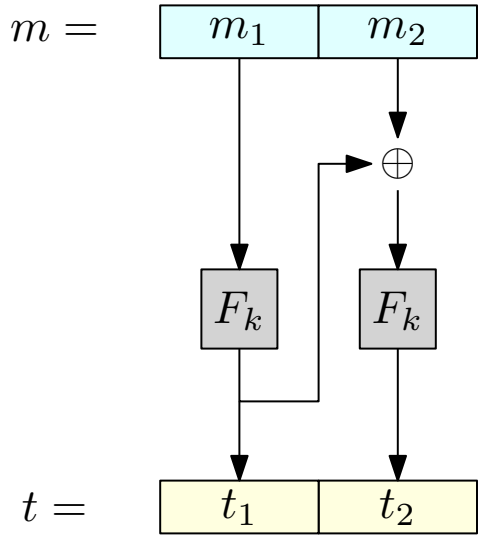

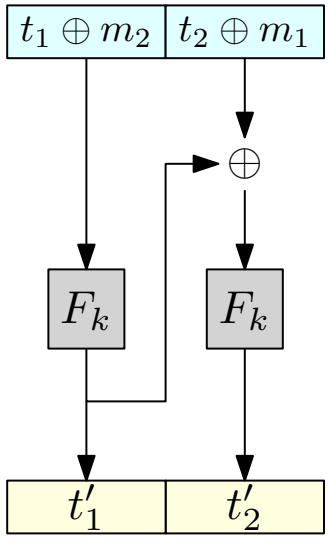

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$t'_1 = F_k(t_1 \oplus m_2) = t_2$

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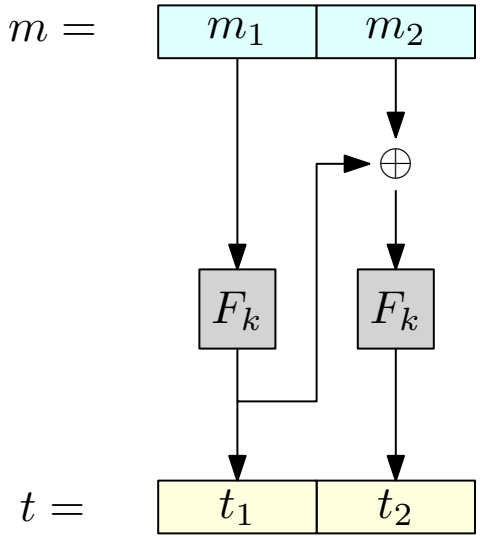

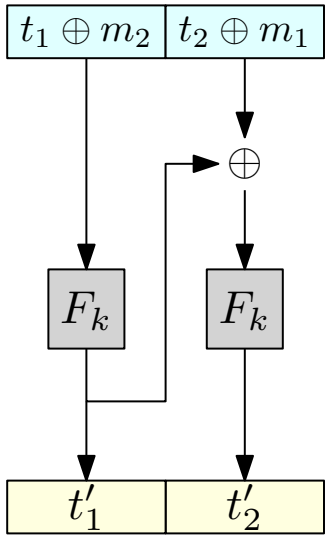

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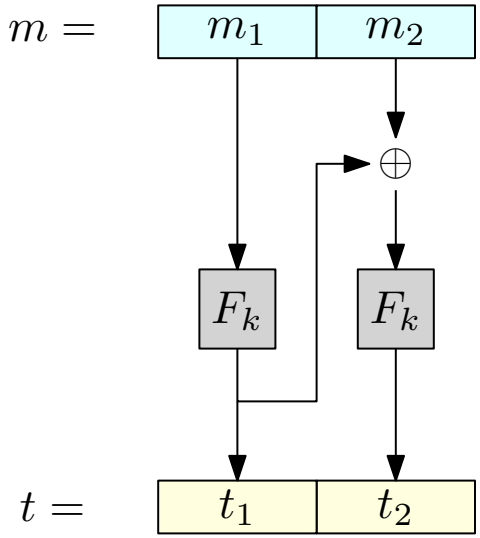

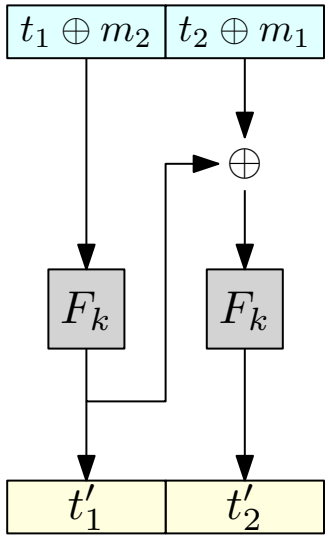

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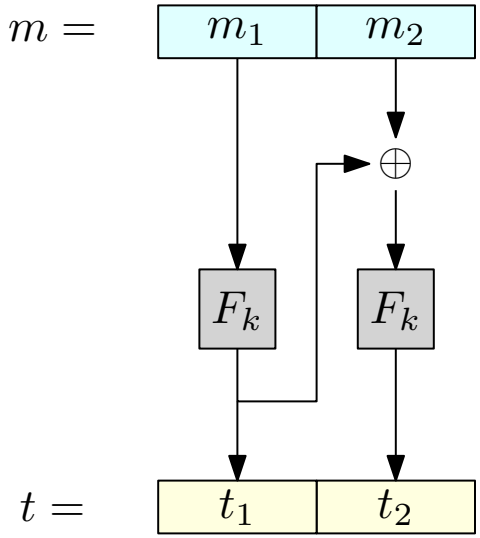

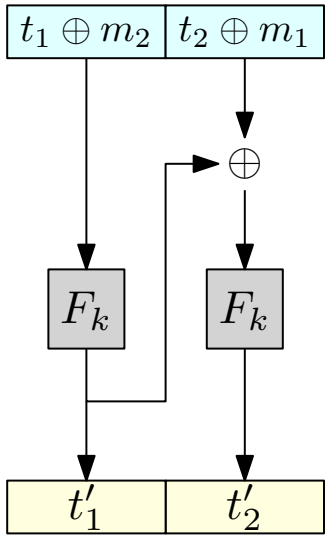

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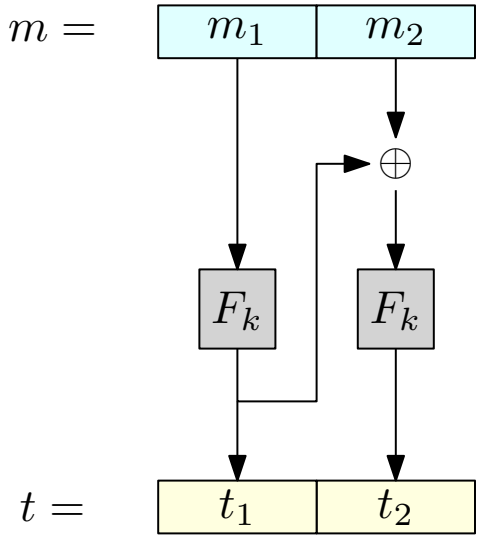

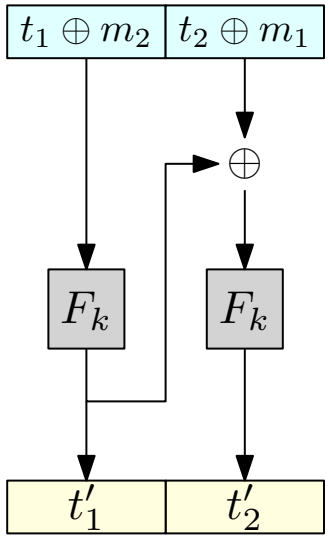

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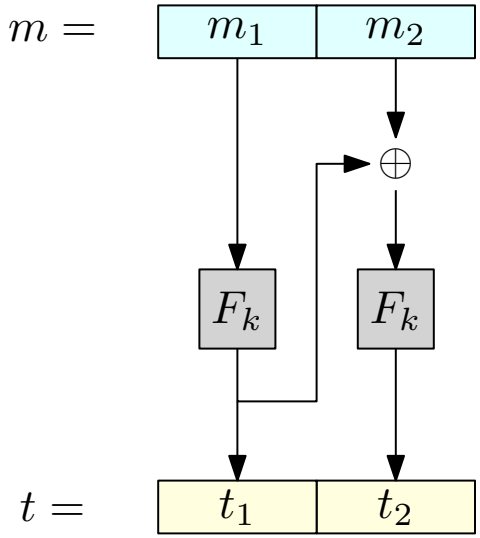

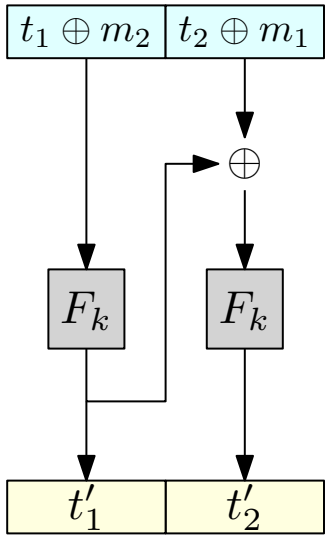

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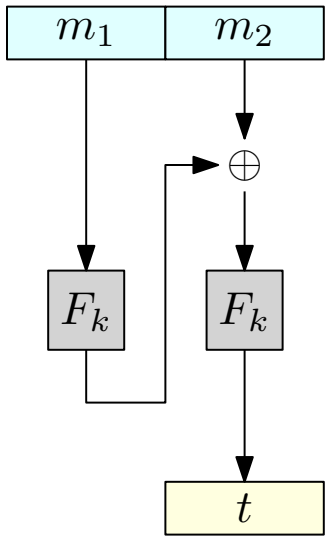

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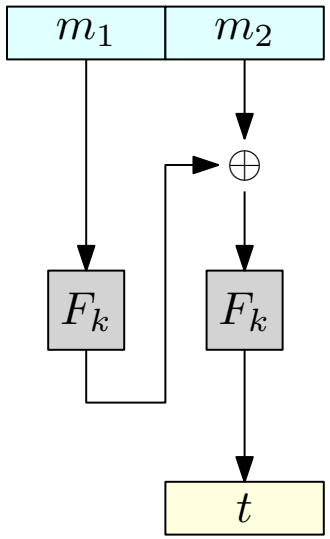

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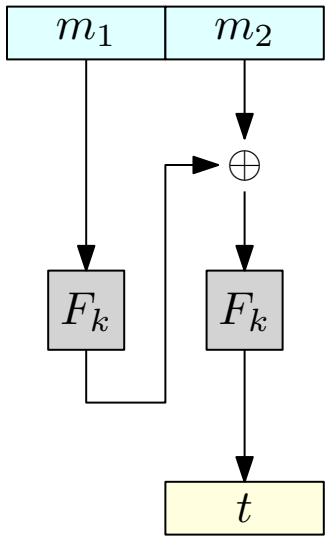

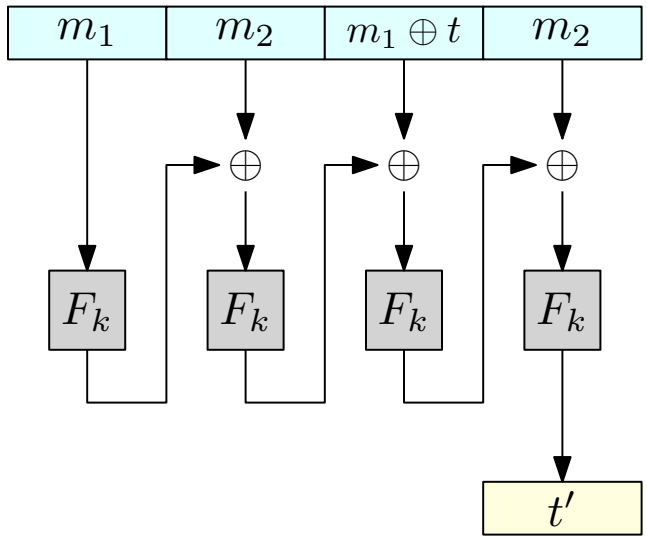

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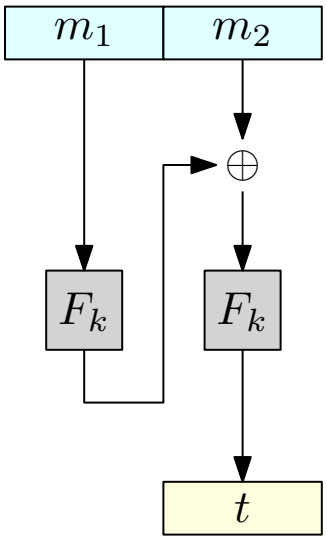

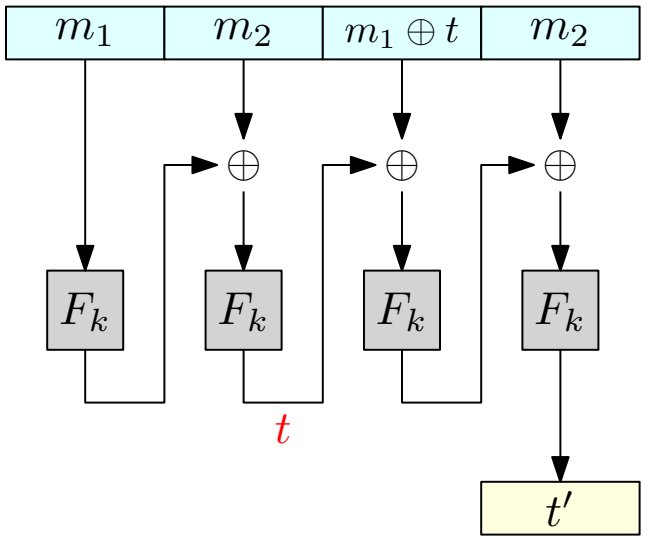

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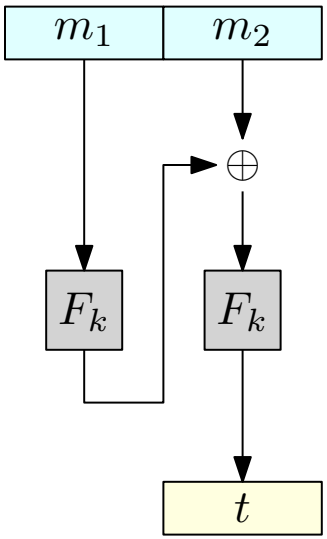

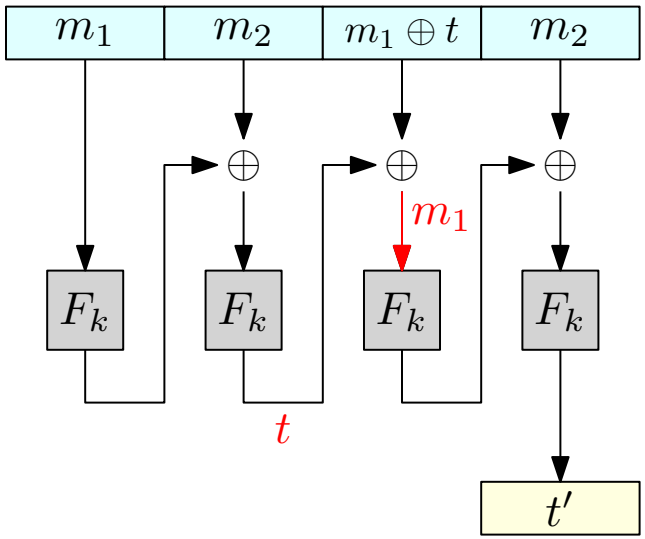

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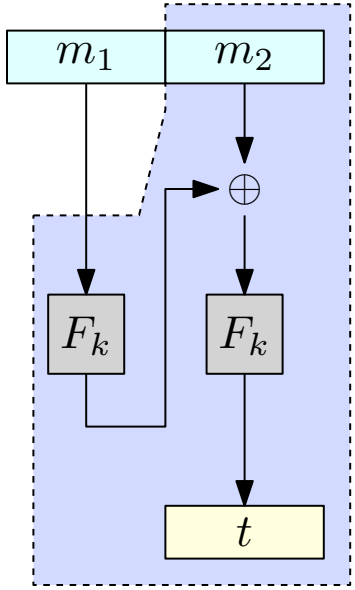

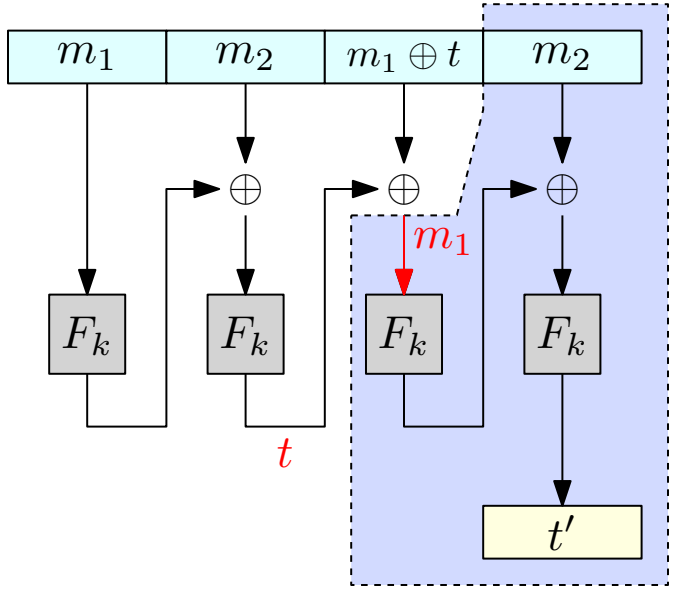

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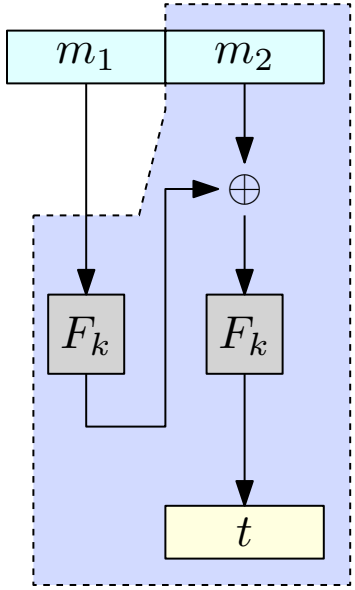

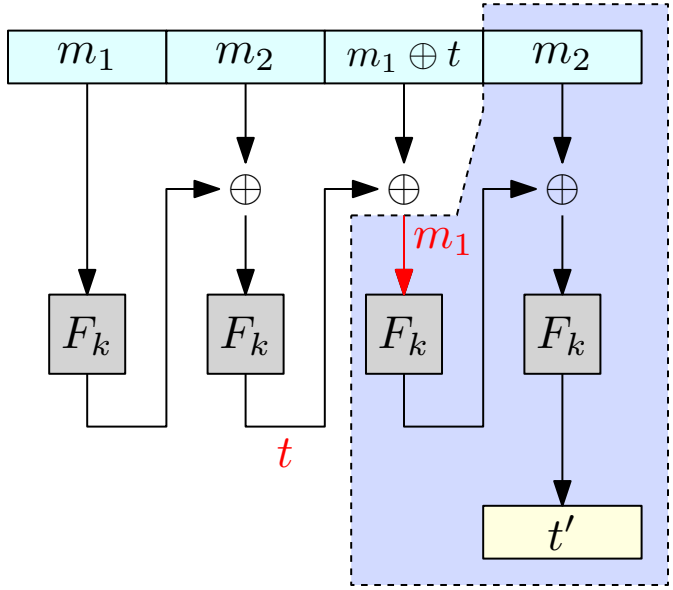

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Basic CBC-MAC can be extended to handle arbitrary-length messages

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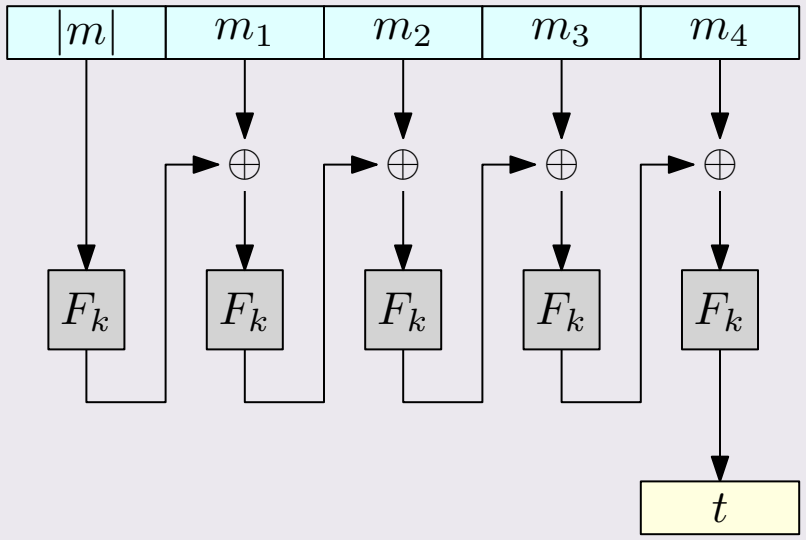
- Encode the message length m as a n-bit string, and **prepend** it to m



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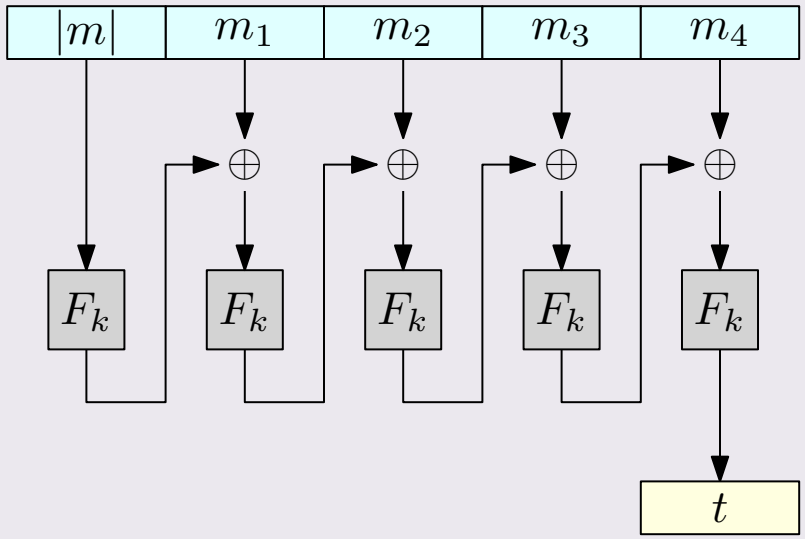

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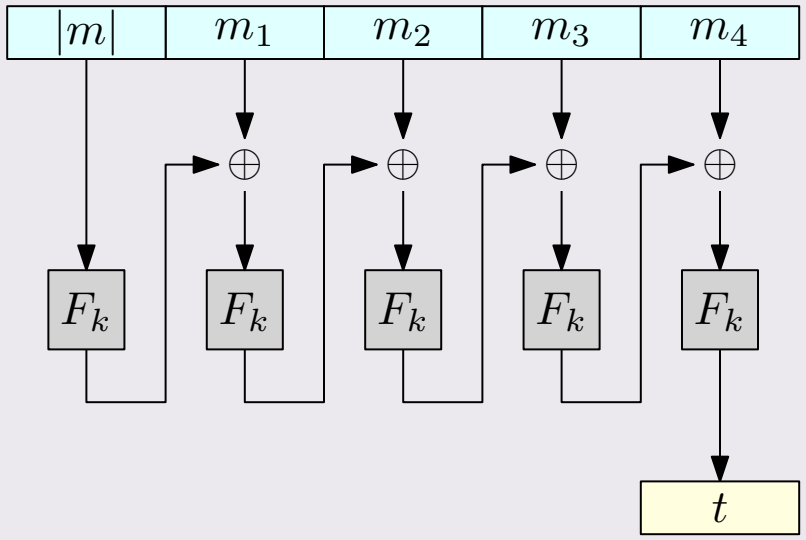

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Note that appending $|m|$ to m is **not secure**

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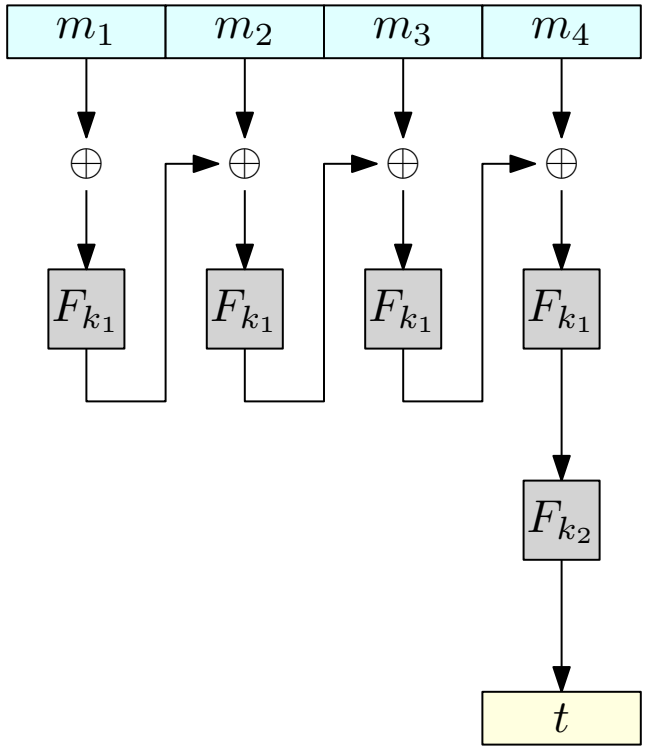

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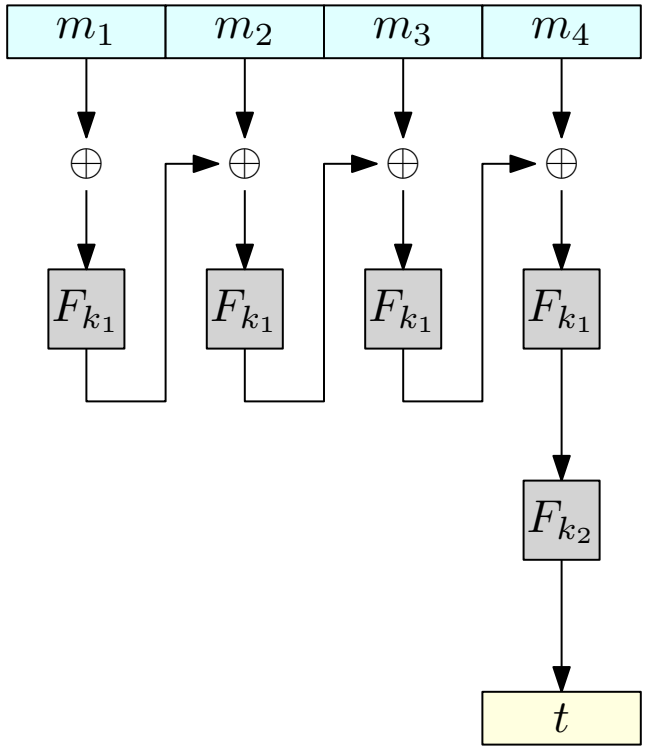

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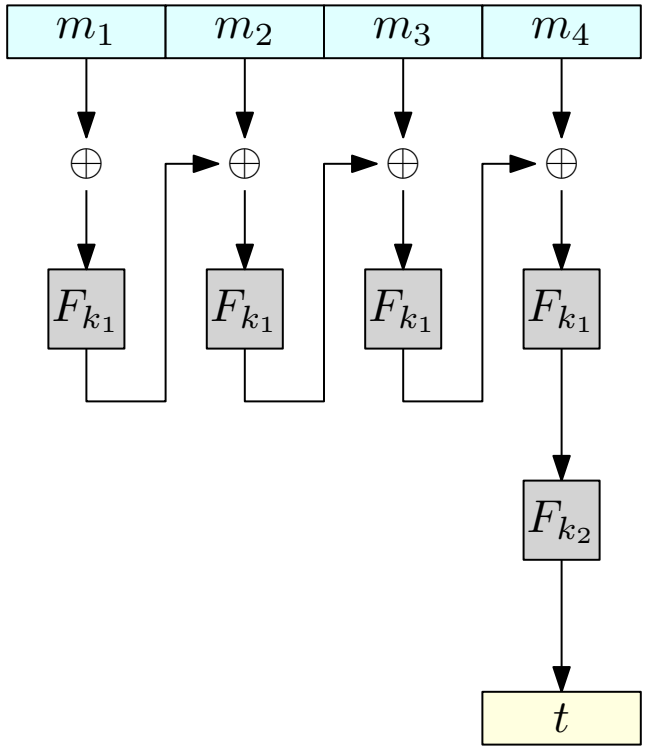

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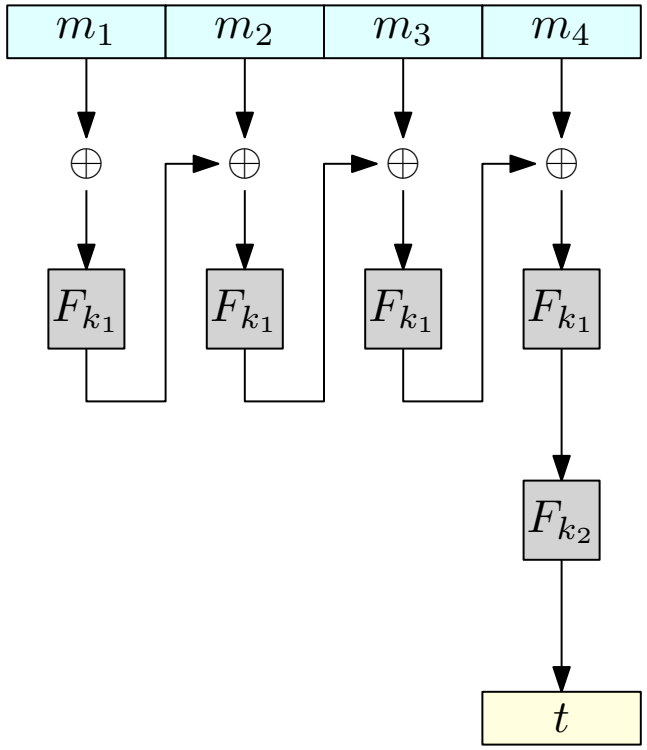

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Advantage: There is no need to know the length of m in advance (Mac_k is a *streaming* algorithm)

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- We can modify our message authentication experiment (and security definition) to account for this

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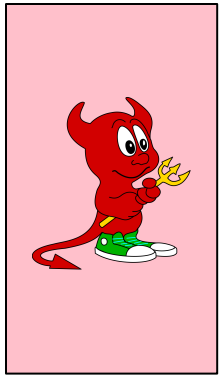

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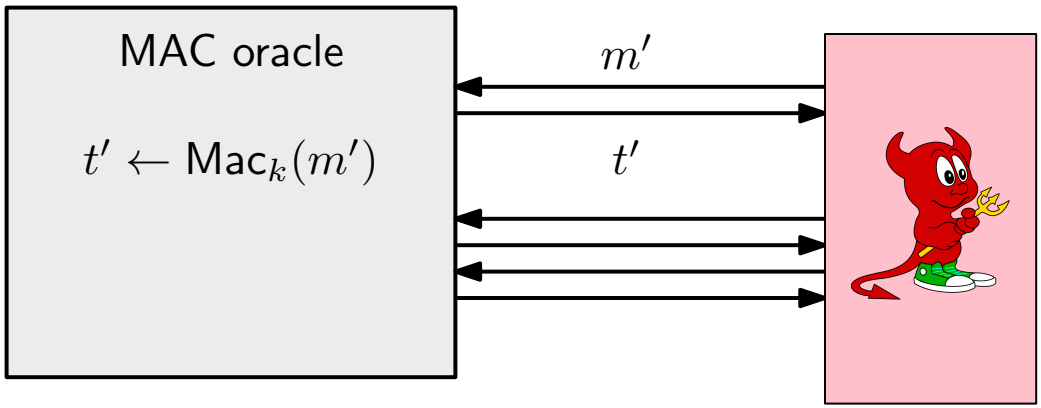

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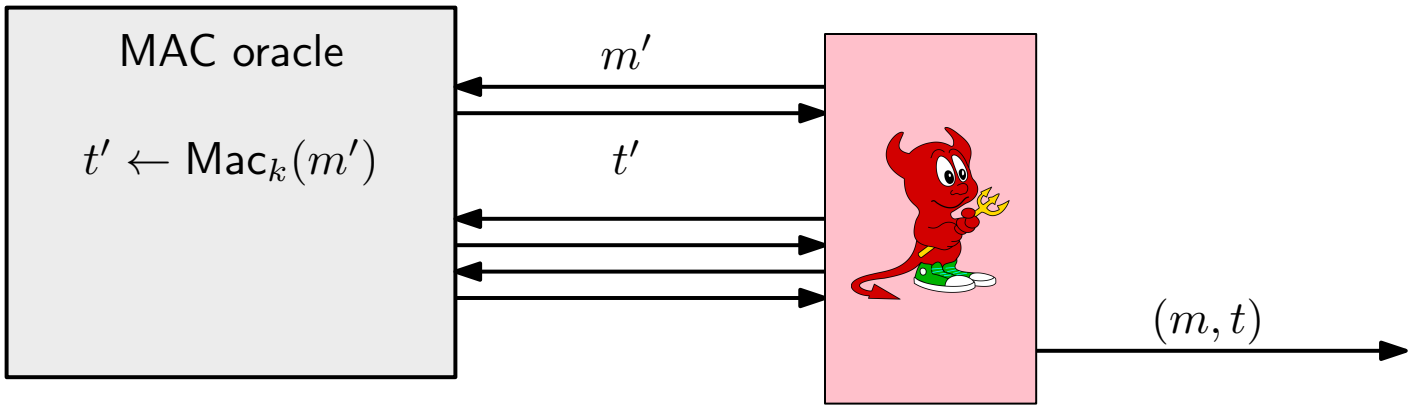

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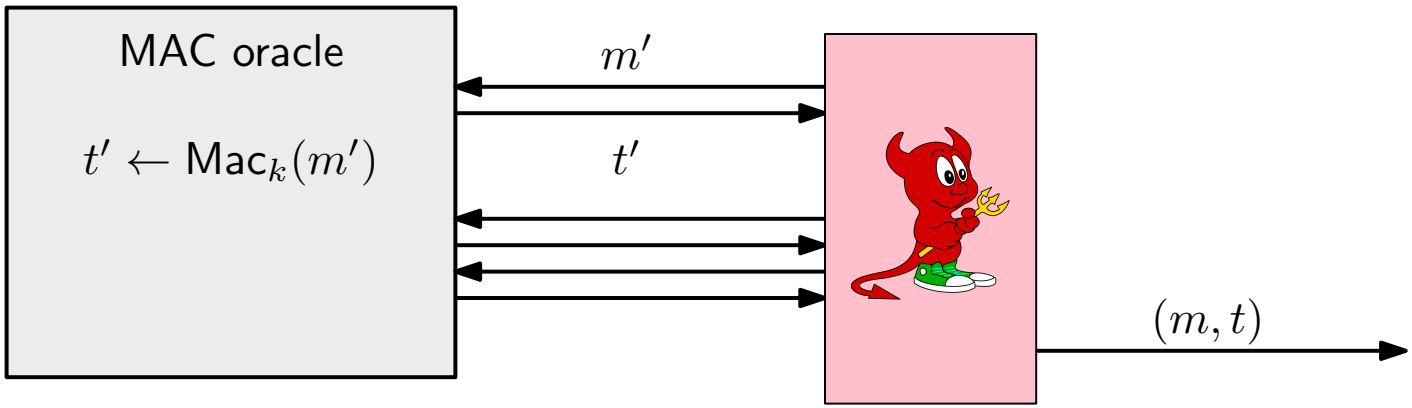
- The outcome of the experiment is 1 if (*) holds and $\mathsf{Vrfy}_k(m, t) = 1$. Otherwise the outcome is 0.



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Good news:

All deterministic secure MACs that use canonical verification are also strongly secure.