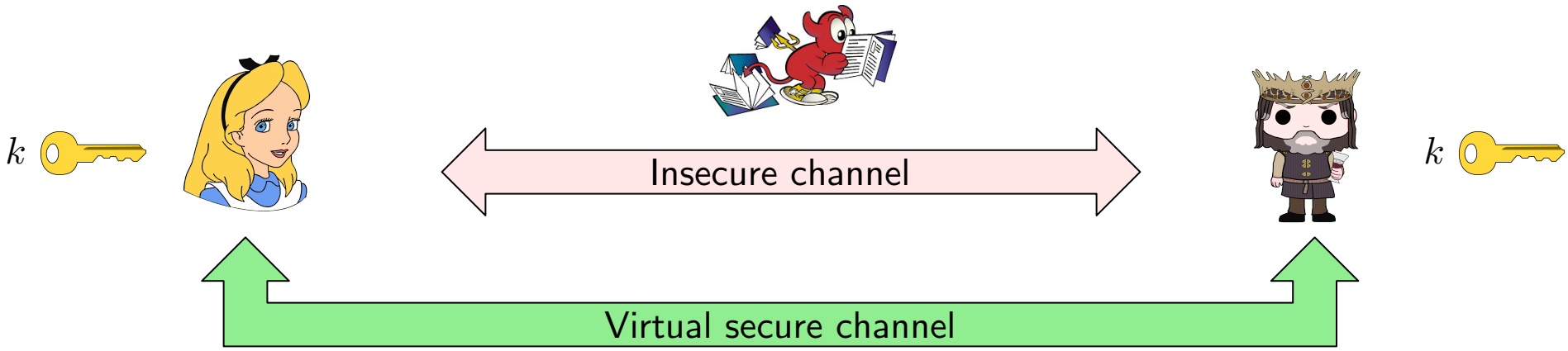
Key Distribution



We have seen how Alice and Bob can use private-key criptography to establish a secure communication channel over an insecure one

To do this they need to share a secret key k

How?

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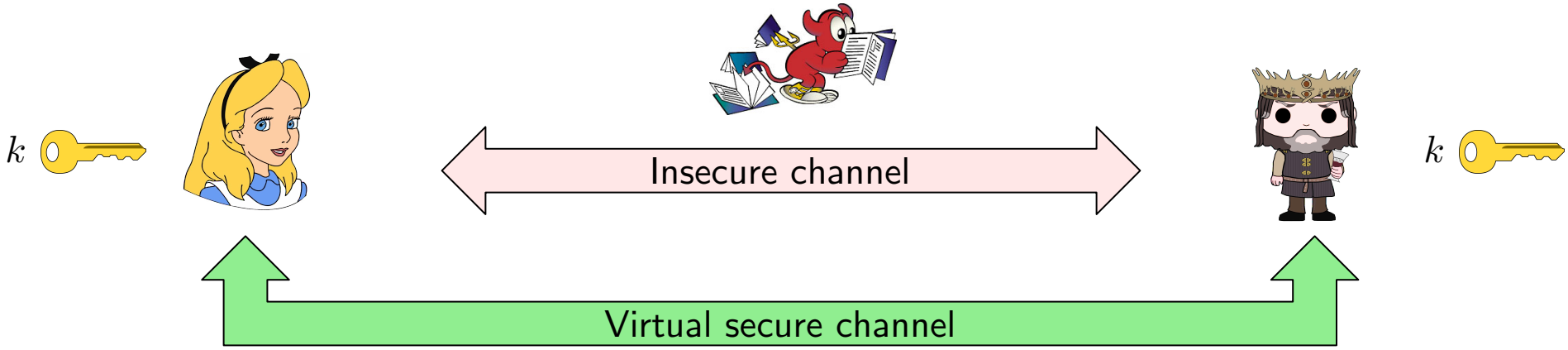

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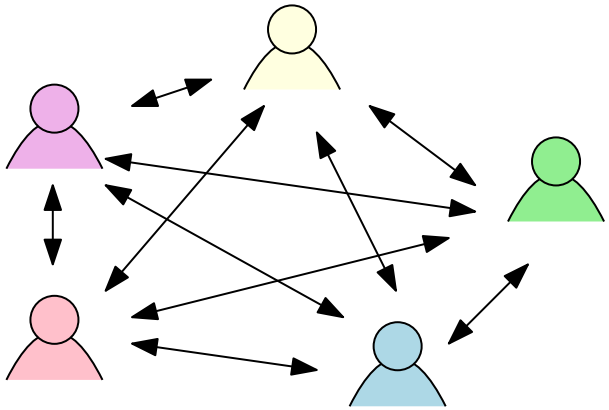
- Alice and Bob can meet in person
- Alice and Bob can use a **trusted** courier

This can be difficult (or even impossible) to do, depending on the situation

Key Management



Imagine a big company with N employees

- Any pair of employee might need to communicate (at some point)
- We want to ensure that the communication is confidential

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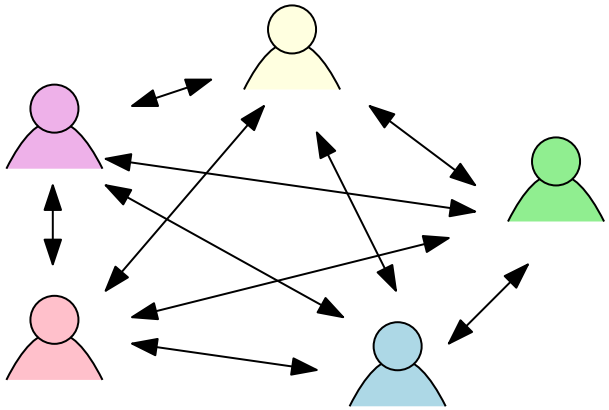

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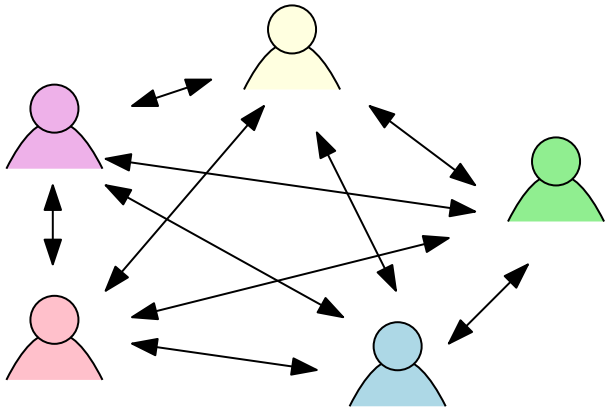

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A possible solution:

- Each pair of users u, v generate their own secret key $k_{u,v}$
- There are $\Theta(N^2)$ keys in total
- If a new user joins the company, $\Theta(N)$ new keys are needed, and they need to be shared privately with each other user

Not great...

A Partial Solution: Key-Distribution Centers ("offline solution")

Suppose that all the entities trust some central authority
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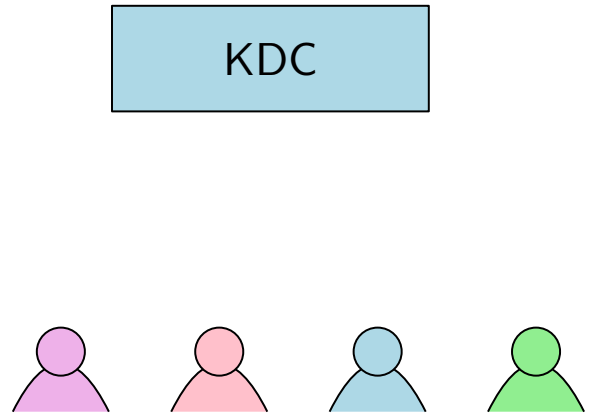

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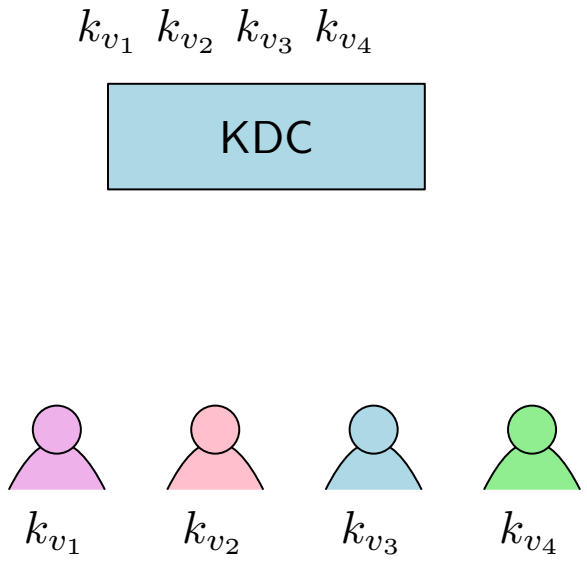

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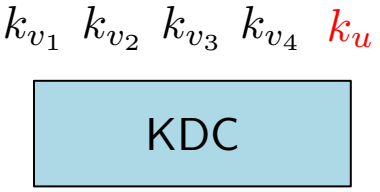

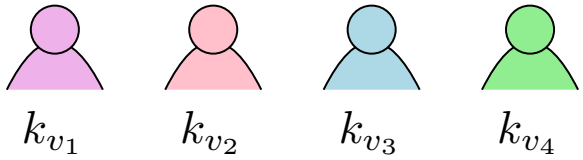

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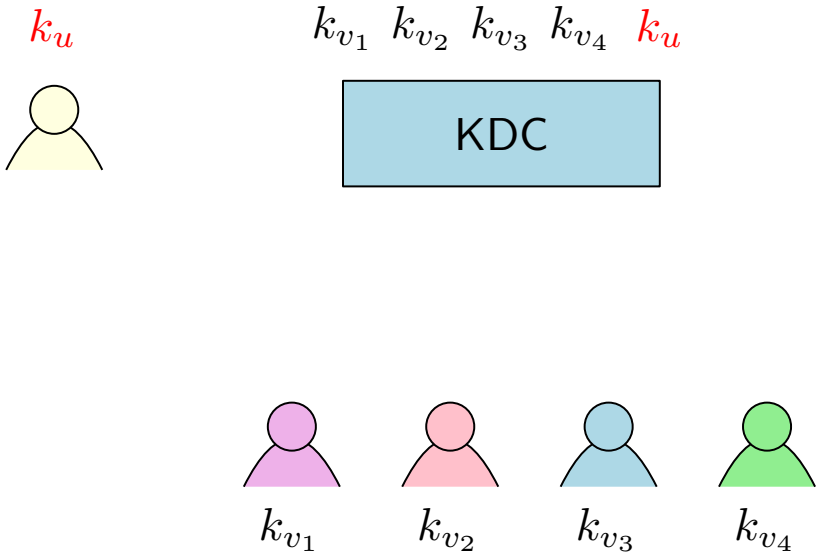

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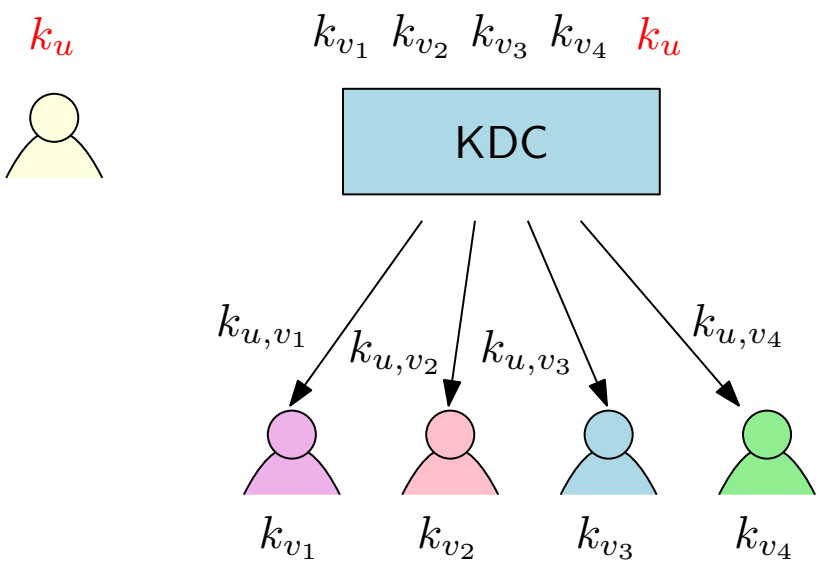

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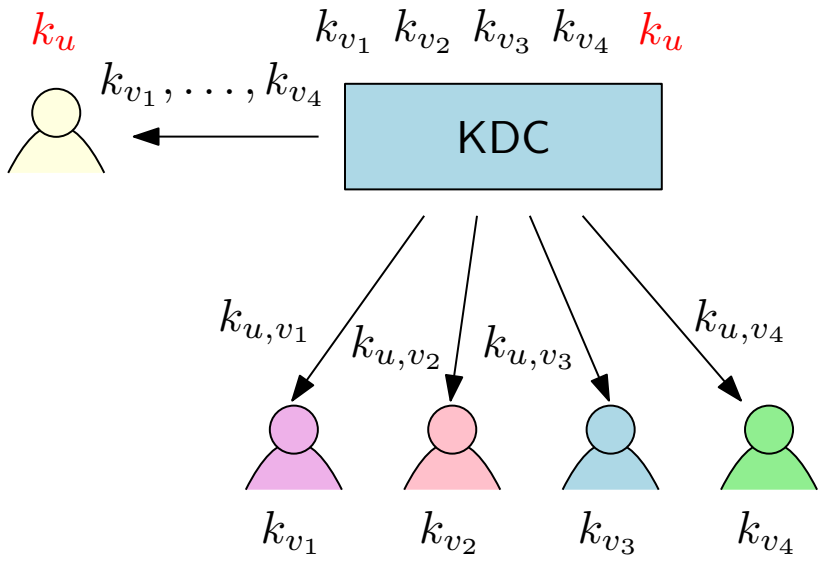

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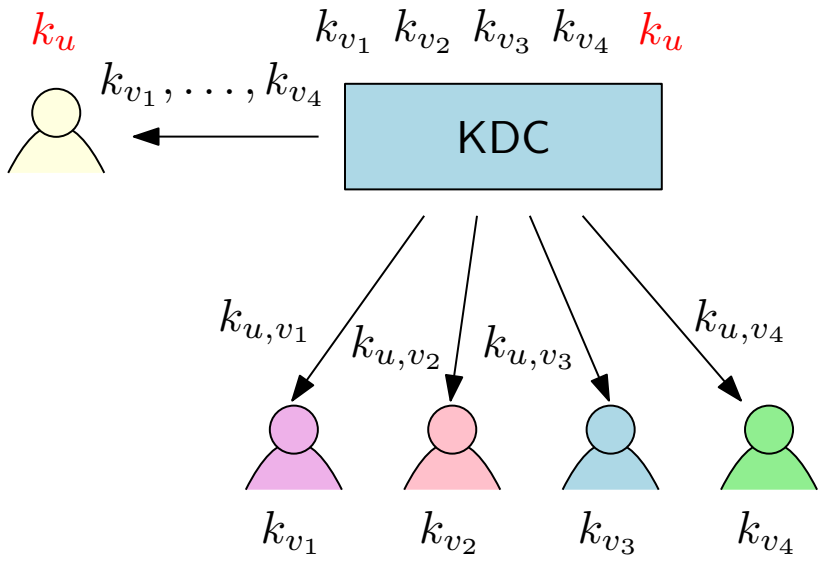

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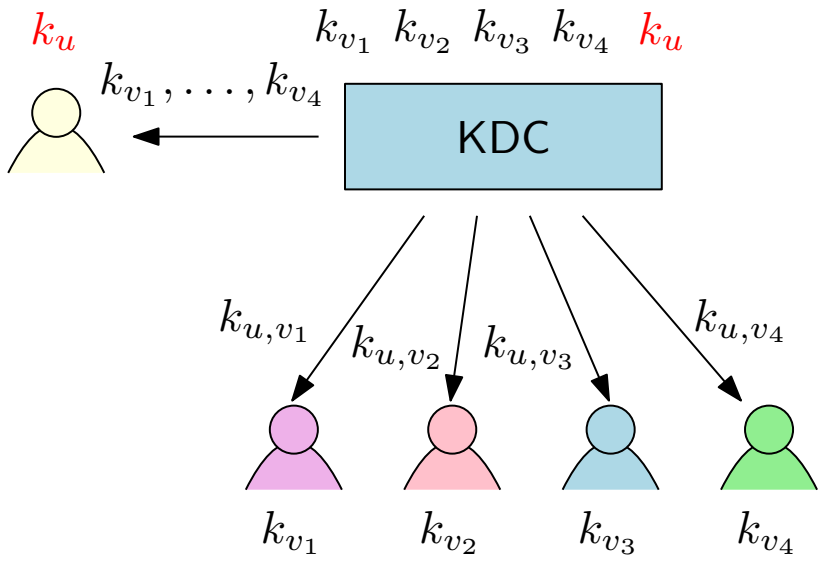

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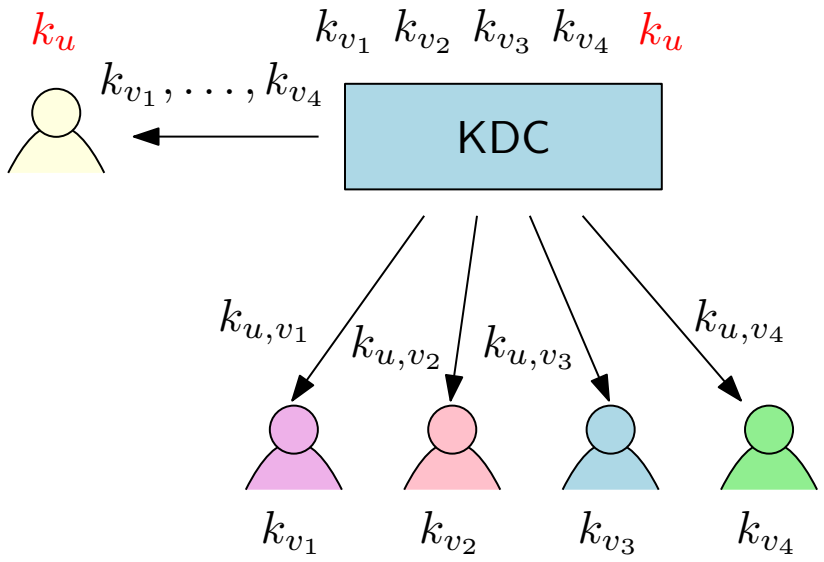

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Con: Still $\Theta(N^2)$ keys overall

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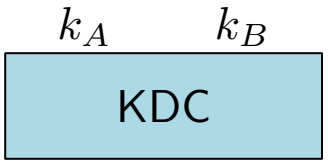

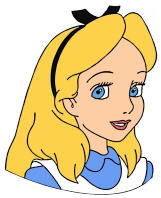

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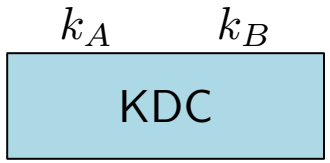

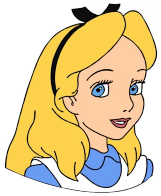

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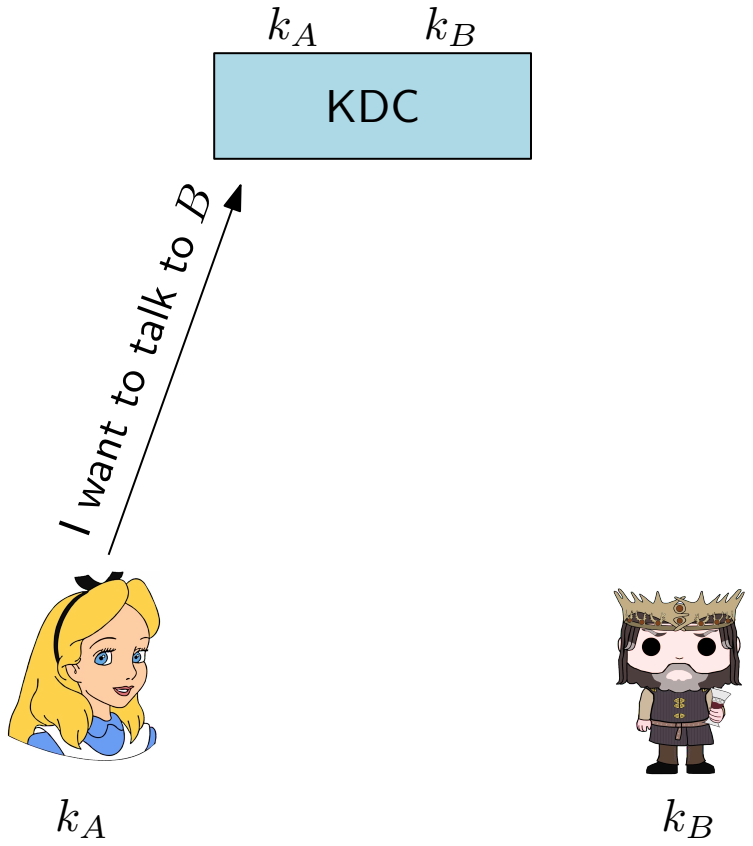

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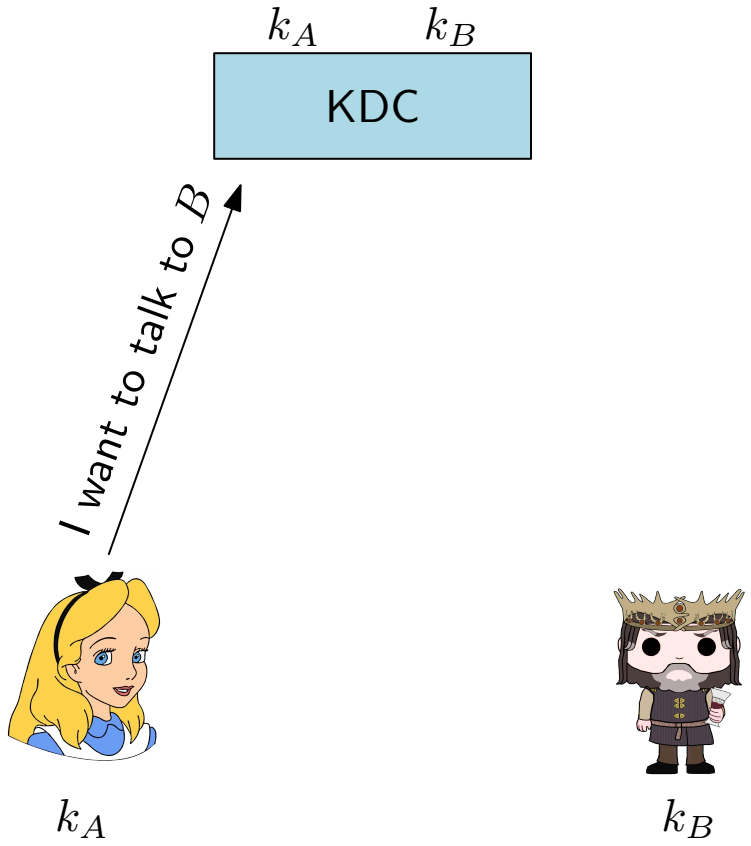

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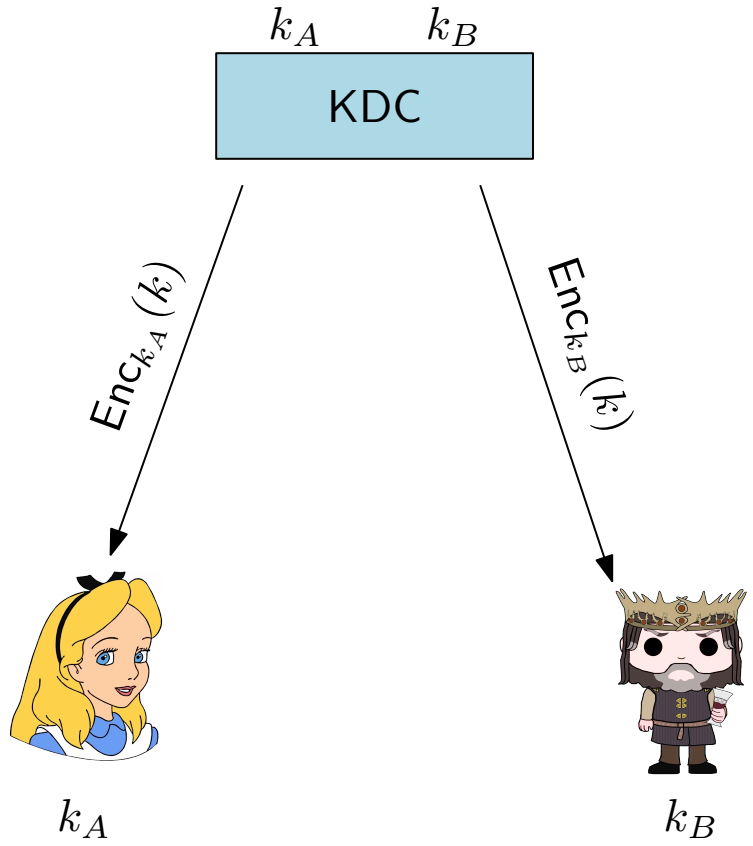

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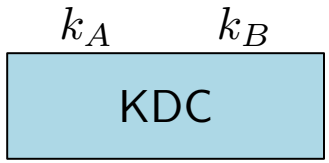

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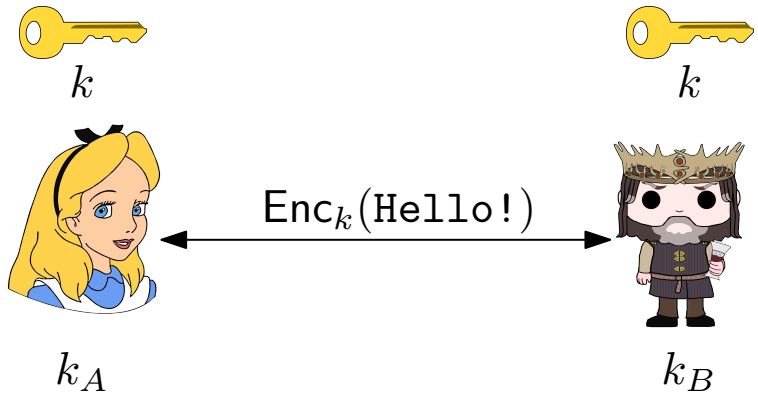

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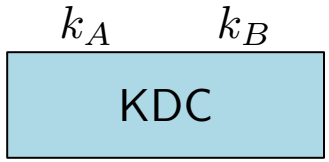

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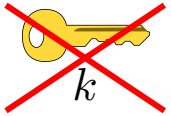

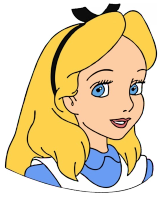

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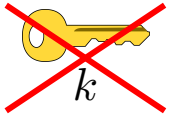

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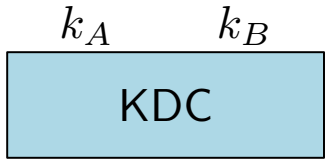

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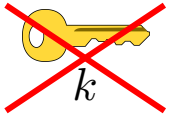

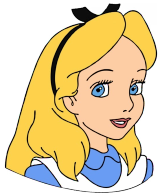

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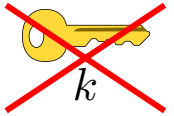

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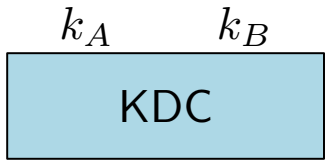

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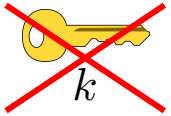

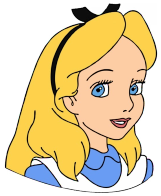

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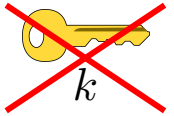

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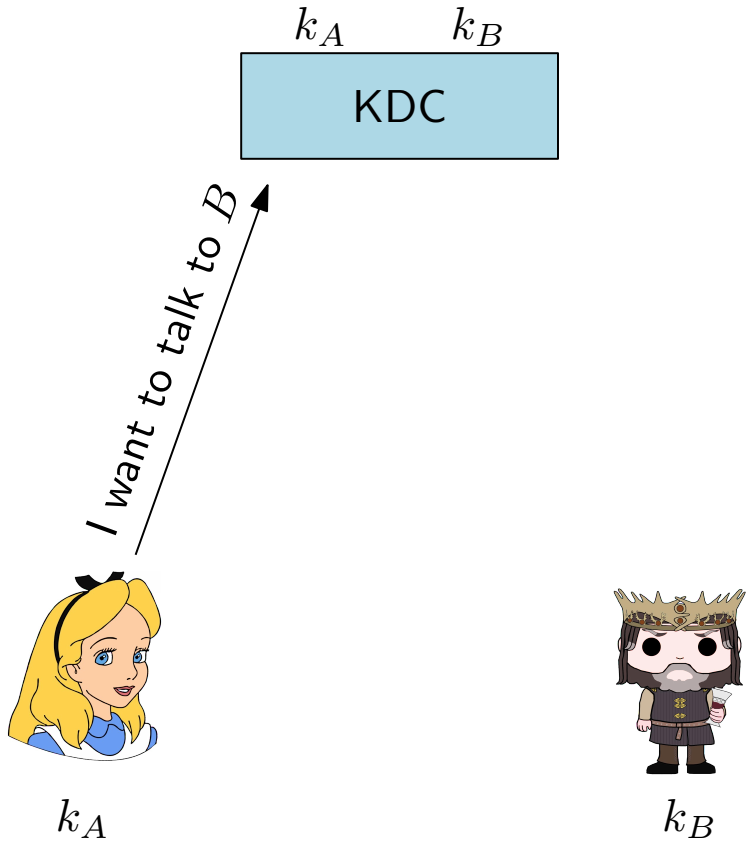
Con: Relies on the KDC being online (KDC can be replicated at the cost of more complexity)

Con: A data breach on the KDC compromises all communications

A Partial Solution: Key-Distribution Centers, an optimization

In practice, a slightly different approach is used

When A contacts the KDC and asks to talk to B...



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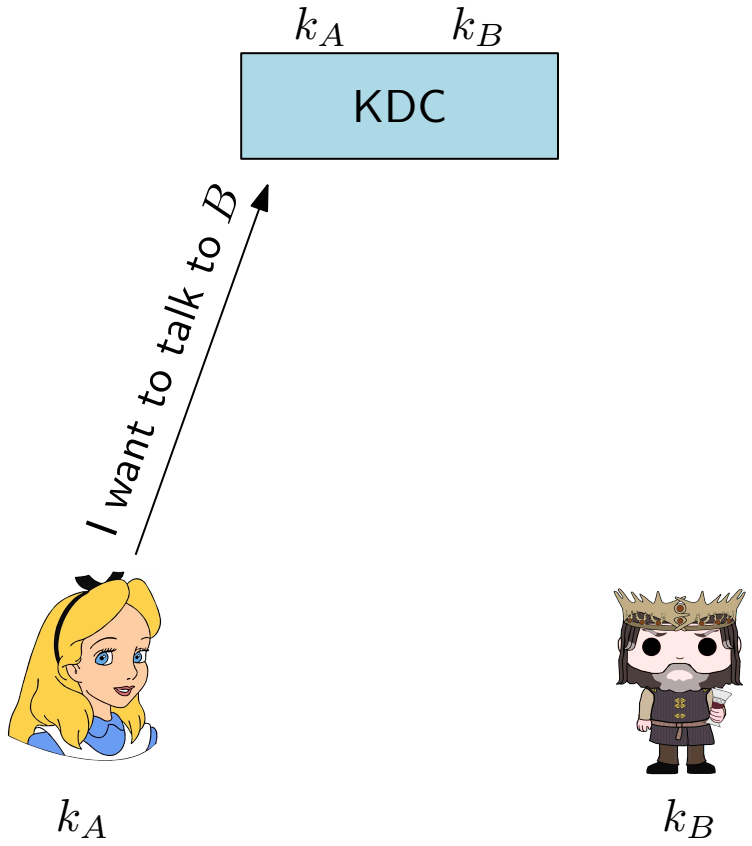

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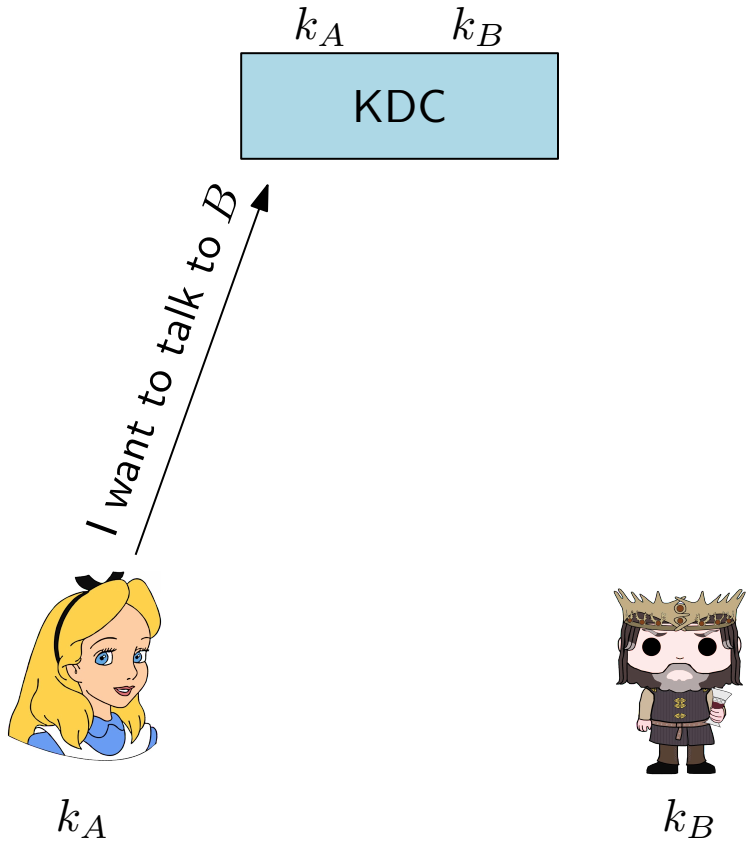

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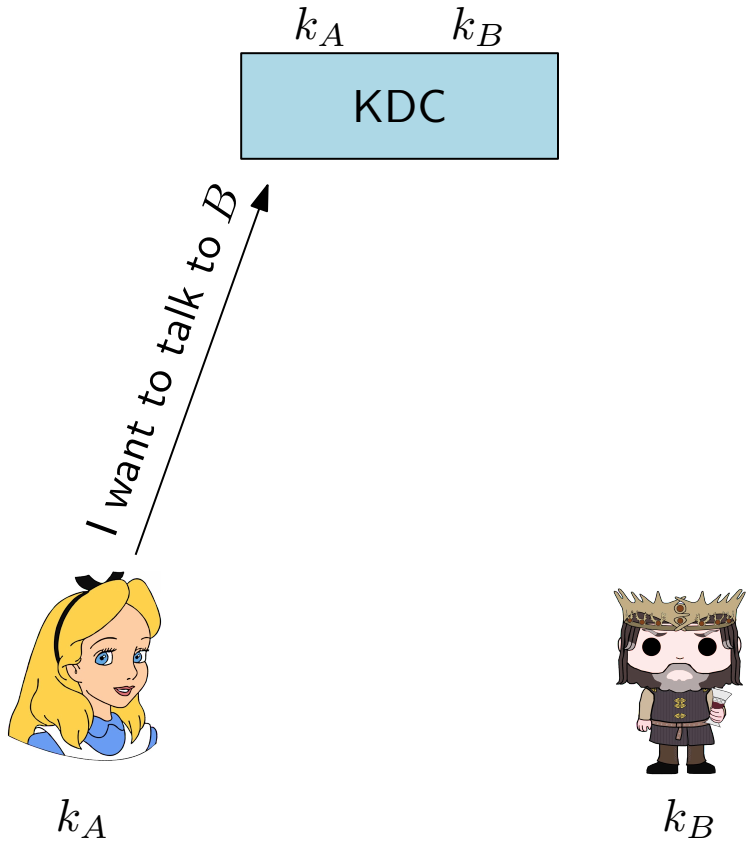

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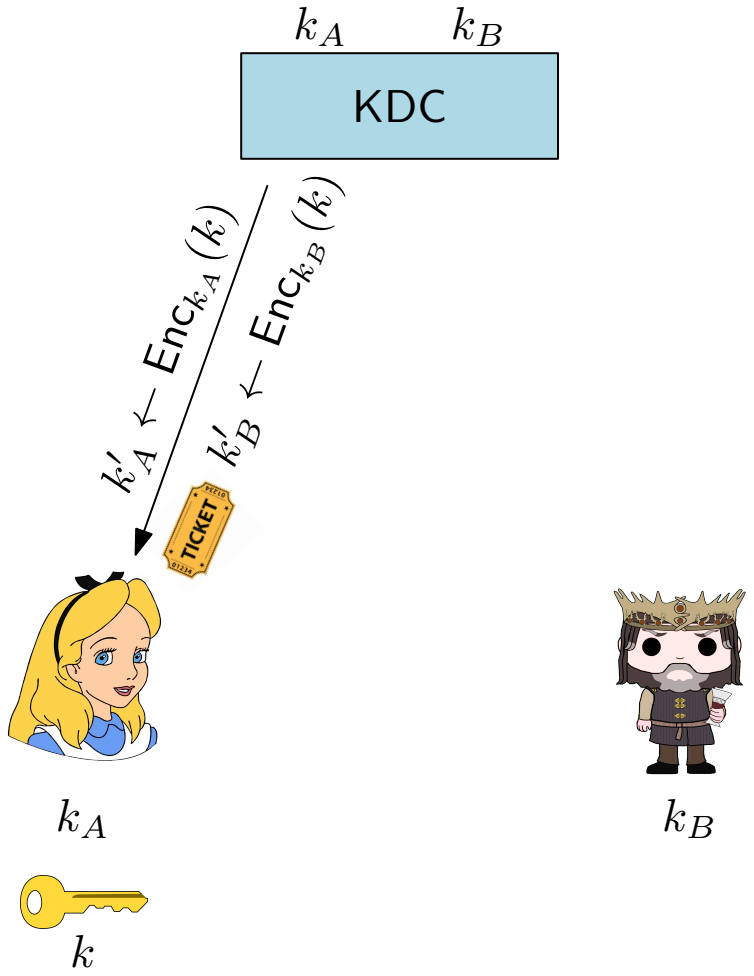

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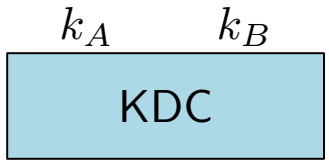

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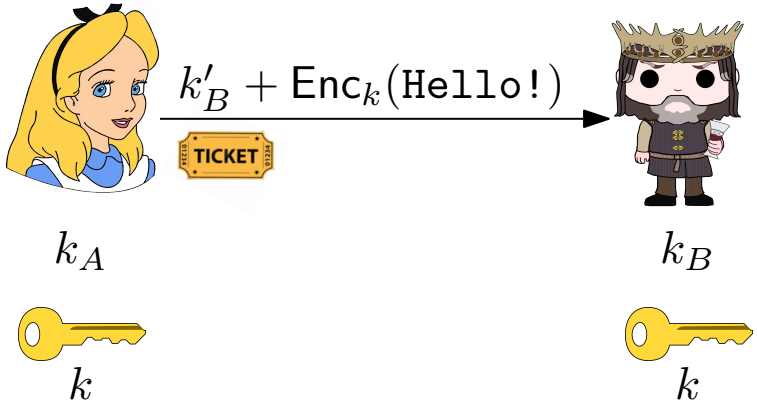

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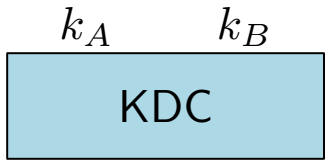

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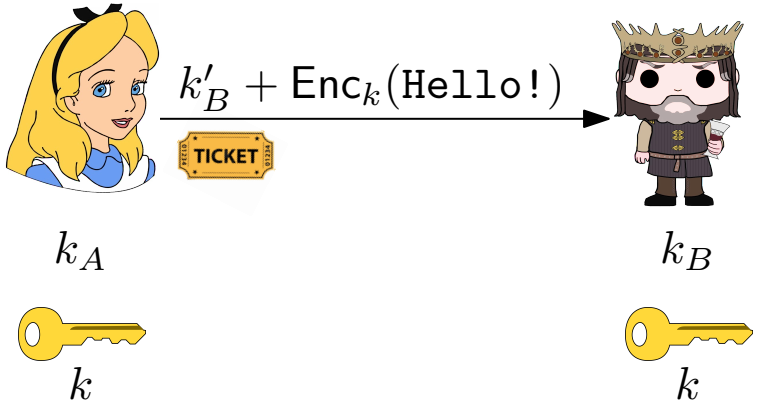

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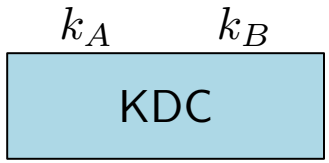

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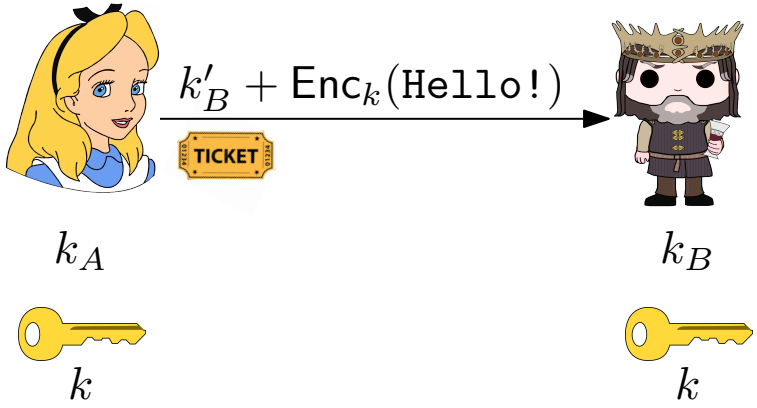

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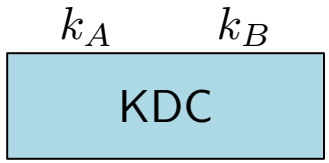

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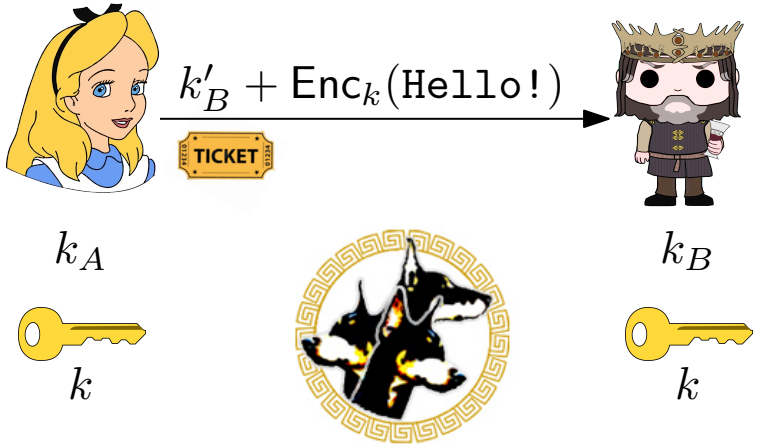
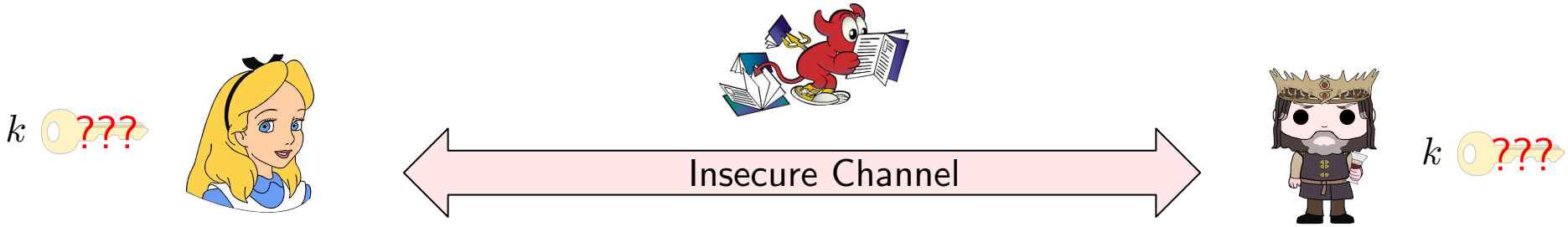
This approach is used, e.g., in the Kerberos protocol.



Open systems

Alice e Bob want to communicate securely over an insecure channel



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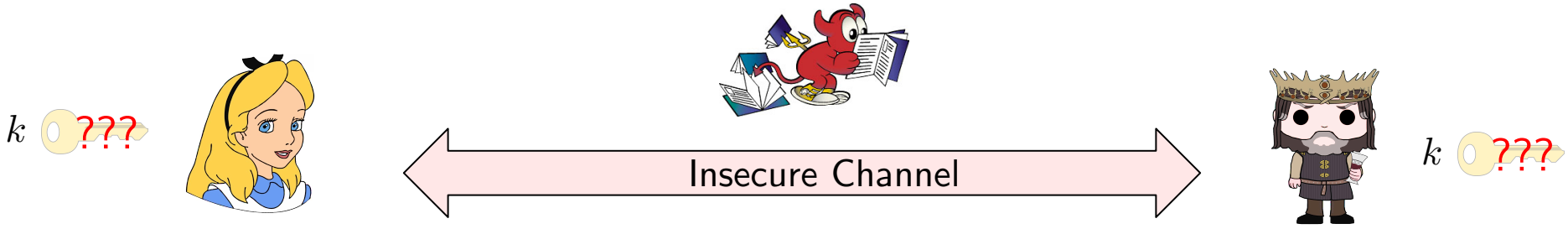

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This happens all the time

- Sending an instant-message to someone for the first time
- Visiting a new website

How do we handle this case?

Key Exchange

Classical private-key cryptography does not offer a satisfactory solution to these problems

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Idea:

- Some problem exhibit asymmetry
- Easy to compute, hard to invert (Candidate examples?)
- Use asymmetry in a clever way, to enable two parties to agree on a secret key over an insecure channel

Key Exchange: Informal Security Definition

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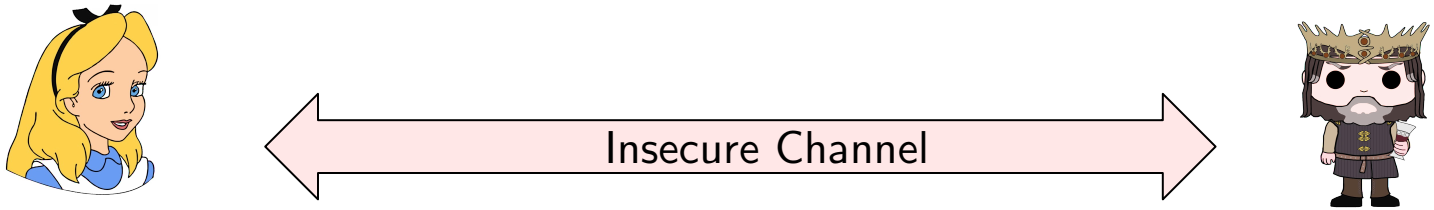

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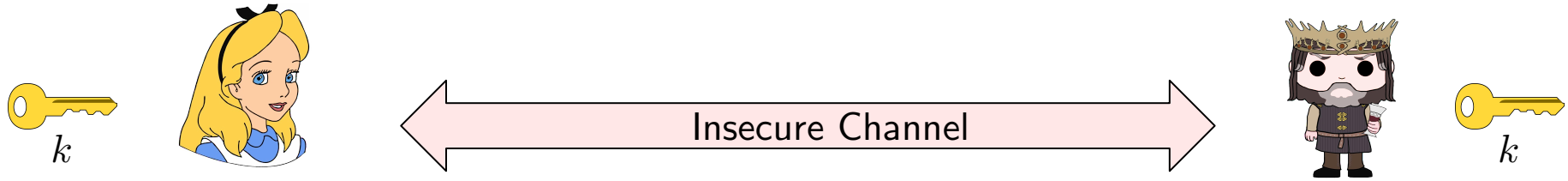

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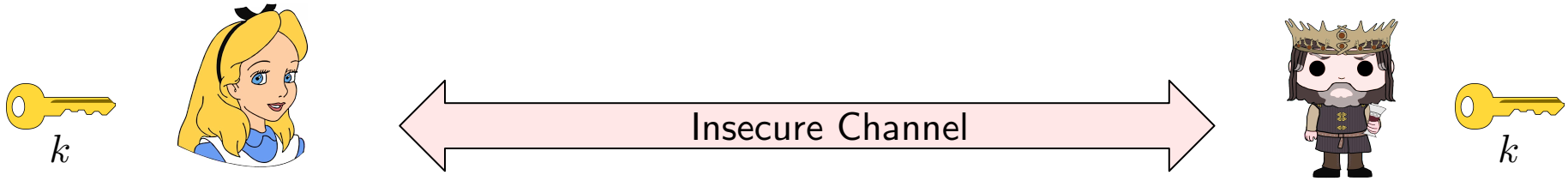

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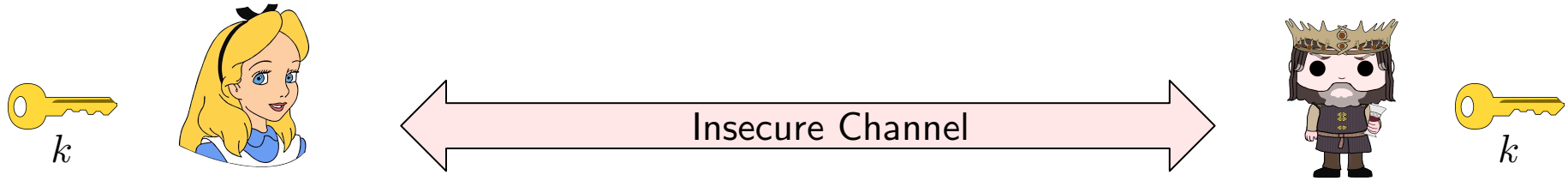

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Security goal (informal): even after observing the transcript, the shared key k should be indistinguishable from a uniform key (to any polynomial-time adversary)

The Key Exchange Experiment

Fix a key exchange protocol Π and an attacker $\mathcal{A}$

We define the **key-exchange experiment** $\mathsf{KE}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n)$ as follows:

- The honest parties run Π using n as the security parameter.
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The outcome of the experiment is defined to be 1 if $b' = b$ and 0 otherwise

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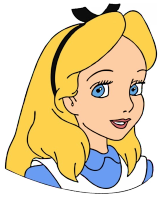

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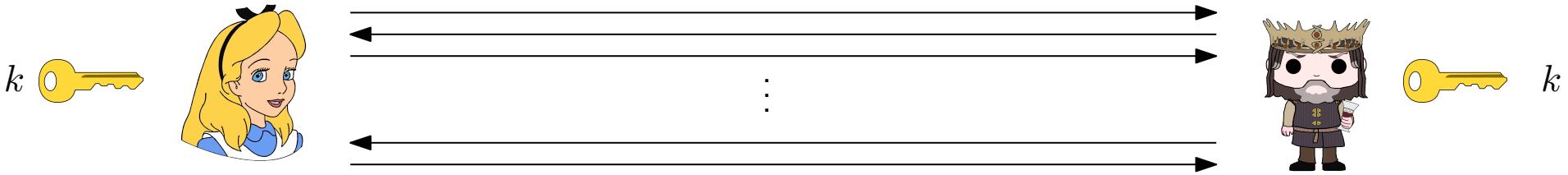

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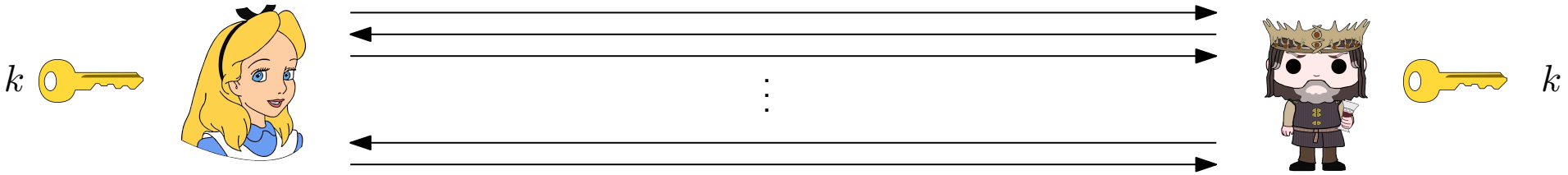

$b \leftarrow \{0,1\}$

$$k' = \begin{cases} k & \text{if } b = 0 \\ \text{random } n\text{-bit string} & \text{if } b = 1 \end{cases}$$

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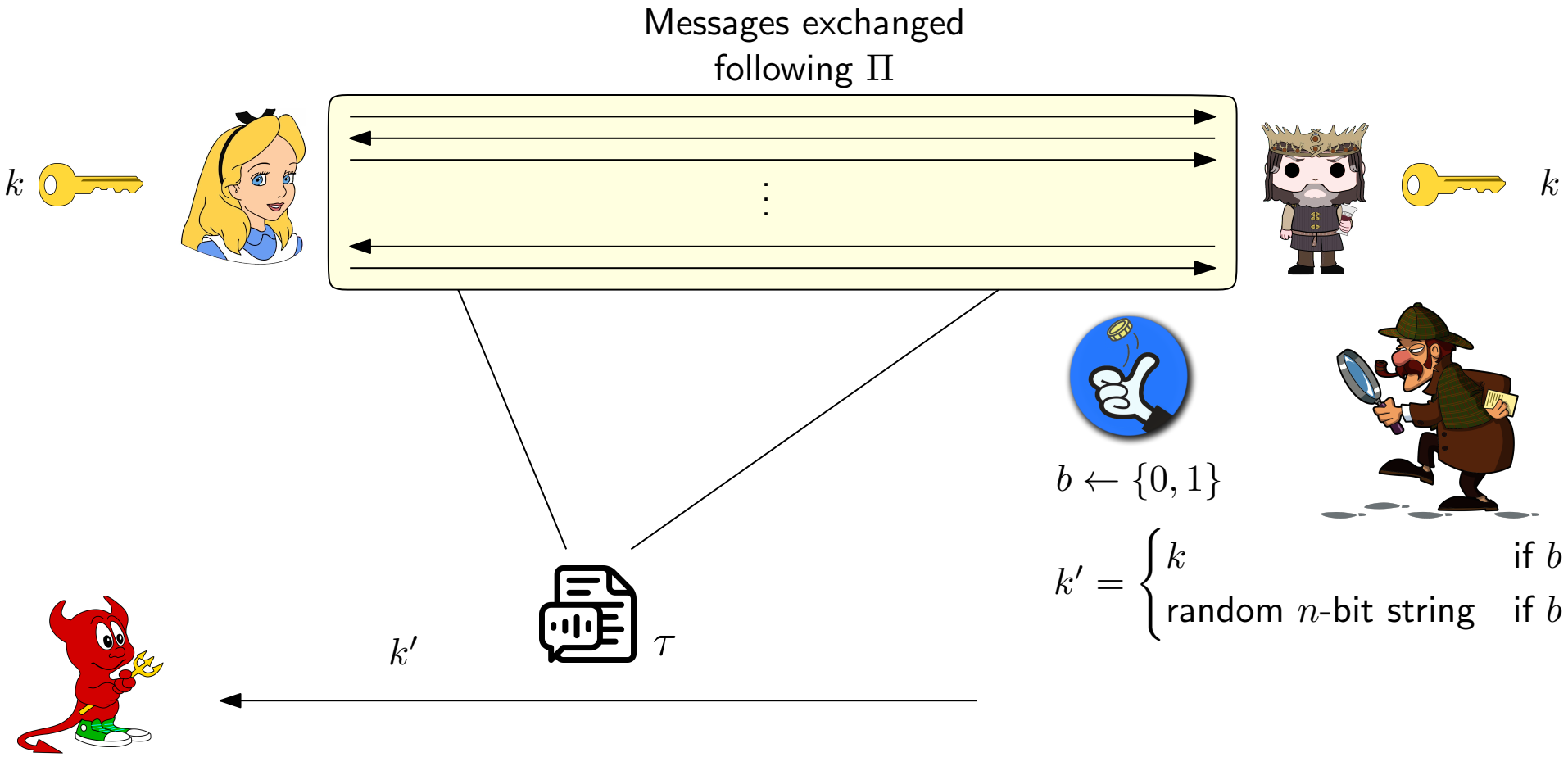

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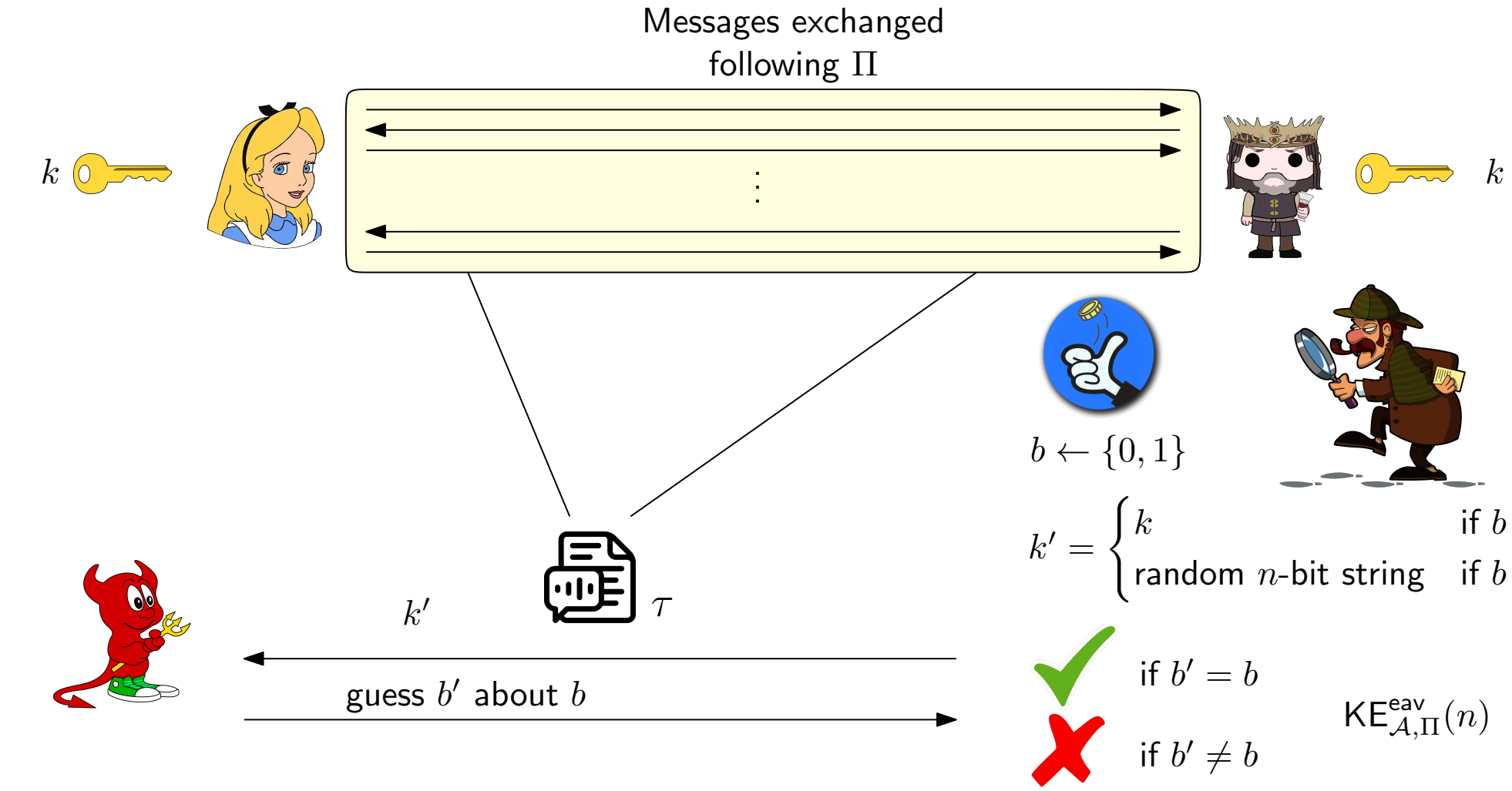

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Notice that being unable to compute k from the transcript τ is not a strong enough security guarantee

- The requirement we impose is **stronger**. Namely k must look just like a random string.
- This is necessary since we are going to use k for private-key cryptography.