Algorithm Design Laboratory with Applications

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Problem: *HDMI cables.*

You work for a company that manufactures HDMI cables of different lengths $\ell_1, \ell_2, \ldots, \ell_k$. Crating a cable of length ℓ_i means cutting a piece of cable of ℓ_i meters from a spool that initially contains n meters of cable and attaching the two HDMI connectors at its endpoints. Each of these connectors costs c Euro cents.

A finished cable of length ℓ_i can be sold for a price of p_i Euro cents. Due to different market demands, prices are not necessarily monotonically increasing with the cable length.

Given n, c, the possible lengths $\ell_1, \ell_2, \ldots, \ell_k$, and the corresponding prices $p_1, p_2, \ldots, p_k$, your goal is to find the best (multi-)set of cables to produce in order to maximize your profit P (i.e., the total revenue from selling the cables minus the overall manufacturing cost).

Input. The input consists of a set of instances, or *test-cases*, of the previous problem. The first line contains the number T of test-cases. The first line of each test-case contains the number n of meters of cable available, the number k of cable lengths than can be produced, and the cost c of a single HDMI connector. The next line contains the k integers $\ell_1, \ldots, \ell_k$. The third and final line of each test case contains the k integers $p_1, \ldots, p_k$.

Output. The output consists of T lines. The i-th line is the answer to the i-th test-case and contains the maximum profit P attainable for the given instance.

Assumptions. $1 \le T \le 10$; $1 \le n \le 2^{20}$; $1 \le k \le 300$; $\forall i = 1, \ldots, k,\ 1 \le \ell_i < 500$ and $1 \le p_i < 500$; $1 \le c \le 100$.

Example.

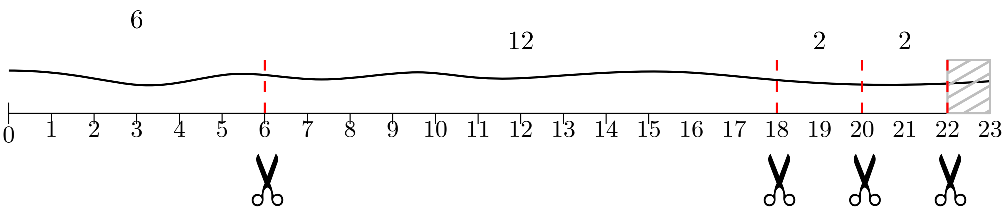

Figure 1: An optimal way to cut a 23 meters long cable when $k = 5$, $c = 2$, $\ell_1 = 6$, $\ell_2 = 12$, $\ell_3 = 2$, $\ell_4 = 3$, $\ell_5 = 8$, and $p_1 = 19$, $p_2 = 54$, $p_3 = 9$, $p_4 = 8$, $p_5 = 22$. Notice that 1 meter of cable is leftover and will not be sold. The total revenue is $19 + 54 + 9 + 9 = 91$ and the manufacturing cost is $4 \cdot 2c = 16$. The profit is $91 - 16 = 75$.

Input (corresponding to the above example):

```
1
23 5 2
6 12 2 3 8
19 54 9 8 22
```

Output:

```
75
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Requirements. Your algorithm should require $O(nk)$ time (with reasonable hidden constants).

Notes. A reasonable implementation should not require more than 1 second for each input file. It is allowed to sell less than n meters of cable.