Algorithm Design Laboratory with Applications

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Problem: *Bookshelf.*

You are shopping for a new bookshelf to organize your collection of $n \in \mathbb{N}^+$ books indexed with the integers in $\{1, \ldots, n\}$. To fit in your room, the bookshelf can only be tall enough to accommodate 3 shelves, that is, 3 rows of books.

The i-th book in your collection ($i = 1, \ldots, n$) has thickness $t_i \in \mathbb{N}^+$, therefore a bookshelf of width W will be able to fit all the books if and only if there is a partition $\mathcal{B} = \{B_1, B_2, B_3\}$ of $\{1, \ldots, n\}$ into 3 sets B_1, B_2, B_3 (corresponding to the books to place on the top, middle, and bottom shelf, respectively) such that $\forall B \in \mathcal{B}, \sum_{i \in B} t_i \leq W$.

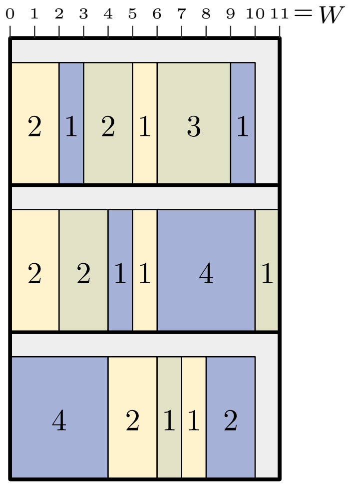
Design an algorithm that, given n and $t_1, \ldots, t_n$, computes the minimum width W of a bookshelf capable of fitting all books.

Input. The input consists of a set of instances, or *test-cases*, of the previous problem. The first line contains the number T of test-cases. Each test case consists of a single line containing the integer n followed by the values $t_1, \ldots, t_n$.

Output. The output consists of T lines. The i-th line is the answer to the i-th test-case and contains the minimum width W of a bookshelf capable of fitting all books.

Assumptions. $1 \leq T \leq 10$; $1 \leq n \leq 2^6$; $\forall i = 1, \ldots, n$, $1 \leq t_i \leq 2^6$.

Example.



Input (corresponding to the above example, book labels represent their thickness):

```
1
17 2 1 3 1 1 2 4 2 1 1 4 2 1 1 1 2 2
```

Output (a possible arrangement is shown in the figure above):

```
11
```

Requirements. Your algorithm should require time $O(n\tau^2)$, where $\tau = \sum_{i=1}^{n} t_i$ (with reasonable hidden constants).

Notes. A reasonable implementation should not require more than 3 seconds for each input file.