

# Algorithm Design Laboratory with Applications

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**Problem:** *Number station.*

A **number station** is a radio station that broadcasts sequences of numbers for espionage operations. You are a secret agent and you just received a transmission with the combination of a safe where vital intelligence is held. The transmission is encrypted and consists of a  $n \times n$  matrix  $A = (a_{i,j})_{i,j}$  of non-negative integers. As per your instructions, you are to recover the combination by counting the number of *non-empty contiguous submatrices* of  $A$  whose sum of elements is odd.

Time is key: design and implement an algorithm to *quickly* compute the safe combination.

**Input.** The input consists of a set of instances, or *test-cases*, of the previous problem. The first line of the input contains the number  $T$  of test-cases. The first line of each test-case is the integer  $n$ . The  $i$ -th of the following  $n$  lines describes the  $i$ -th row of  $A$  and contains the  $n$  integers  $a_{i,1}, \dots, a_{i,n}$  separated by a space.

**Output.** The output consists of  $T$  lines, each containing a single integer. The  $i$ -th line is the answer to the  $i$ -th test-case and is the number of quadruples of integers  $(h, k, x, y)$  with  $1 \leq h \leq k \leq n$  and  $1 \leq x \leq y \leq n$  such that  $\sum_{i=h}^k \sum_{j=x}^y a_{i,j}$  is odd.

**Example.** The safe combination associated with the matrix

$$A = \begin{bmatrix} 1 & 6 & 5 \\ 1 & 4 & 8 \\ 0 & 2 & 4 \end{bmatrix}$$

is 16, since  $A$  has 16 non-empty contiguous submatrices whose elements sum to an odd number.

Namely:  $[1]$ ,  $[5]$ ,  $[1]$ ,  $[1 \ 6]$ ,  $[6 \ 5]$ ,  $[1 \ 4]$ ,  $[1 \ 4 \ 8]$ ,  $\begin{bmatrix} 5 \\ 8 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 5 \\ 8 \\ 4 \end{bmatrix}$ ,  $\begin{bmatrix} 6 \ 5 \\ 4 \ 8 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \ 4 \\ 0 \ 2 \end{bmatrix}$ ,

$\begin{bmatrix} 1 \ 6 \ 5 \\ 1 \ 4 \ 8 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \ 4 \ 8 \\ 0 \ 2 \ 4 \end{bmatrix}$ ,  $\begin{bmatrix} 6 \ 5 \\ 4 \ 8 \\ 2 \ 4 \end{bmatrix}$ , and  $A$  itself. Notice that the submatrix  $[1]$  is counted twice since it appears two times in  $A$ .

*Input (corresponding to matrix  $A$  in the above example):*

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1
3
1 6 5
1 4 8
0 2 4
```

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*Output:*

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16
```

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**Assumptions.**  $1 \leq T \leq 10$ ;  $1 \leq n \leq 512$ ;  $\forall i, j = 1 \dots, n, 0 \leq a_{i,j} \leq 1024$ .

**Requirements.** Your algorithm must have an asymptotic time complexity of  $O(n^3)$ .

**Notes.** A reasonable implementation should not require more than 1 second for each input file.