Algorithm Design Laboratory with Applications

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Problem: *Postal service.*

You are employed in the post office responsible for delivering shipments to the city of Algoland. Shipments consists of one or more parcels, each with its own weight, and are delivered using one or more of the $n > 0$ available delivery trucks $T_1, \dots, T_n$, where truck T_i is capable of carrying a maximum weight of $C_i \in \mathbb{N}^+$ Grahams (the unit of weight of Algoland).

Unlike the nearby post office of Sloppyland (a slow office with lazy employees and a yellow logo), the postal service of Algoland is very efficient: a shipment is always delivered in the shortest possible time.

A pesky computer scientist decides to test the system by shipping his entire collection of huge and heavy anvils to a friend. The shipment consists of $m > 0$ parcels $P_1, \dots, P_m$, where P_j weighs $w_j \in \mathbb{N}^+$ Grahams. A truck T_i can carry a parcel P_j if and only if $w_i \le C_j$. Due to the complicated road network of the city, the time required to travel from the post office (where the parcels are currently stored) to the recipient's address is of 5 minutes, while the time needed to travel back to the post office is of 3 minutes. Moreover, due to the humongous size of the anvils, each truck can accommodate at most one parcel per trip (but it can perform multiple trips).

Your want to design an algorithm to compute a schedule that delivers all of $P_1, \dots, P_m$ to the recipient's address in the minimum amount of time. Packages can be delivered in any order and you should minimize the time at which the last package to be delivered reaches the recipient.

Provide a formal proof of the correctness of your algorithm.

Input.

The input consists of a set of instances, or *test-cases*, of the previous problem. The first line of the input contains the number T of test-cases. A test case is described by 3 lines. The first line of each test-case contains n and m, the second line contains the n integers $C_1, \dots, C_n$, and the third and final line contains the m integers $w_1, \dots, w_m$.

Output. The output consists of T lines, each corresponding to a test-case. The i-th of the lines contains a single integer M_i representing the shortest amount of time needed to deliver all packages, in minutes.

Example.

Input:

```
1
3 5
12 7 15
14 8 13 5 10
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Output:

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13
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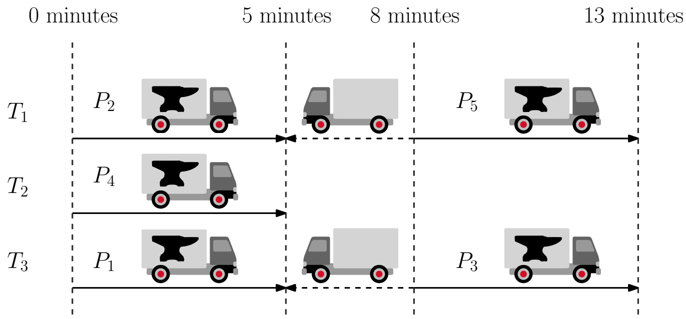
A possible schedule achieving the optimal delivery time of 13 minutes is the following:

- Deliver (simultaneously) P_1 with T_3, P_2 with T_1, P_4 with T_2. This requires 5 minutes.
- Drive (simultaneously) T_1 and T_3 back to the post office. This requires 3 minutes.
- Deliver (simultaneously) P_3 with T_3 and P_5 with T_1. This requires 5 minutes.

Assumptions. $1 \le T < 10$; $\quad 1 \le n < 2^{16}$; $\quad 1 \le m \le 2^{21}$; $\quad \max_{j=1,\dots,m} w_j \le \max_{i=1,\dots,n} C_i \le 2^{10}$.



Requirements. Your algorithm must have an asymptotic time complexity of $O((m+n)\log(m+n))$.

Notes. A reasonable implementation should not require more than 1 second for each input file. Trucks are not required to travel back to the post office for the delivery to be considered completed.