## Algorithm Design Laboratory with Applications

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## Problem: Travelling salesman.

A long highway traverses $n$ cities, indexed from 1 to $n$. ACME Inc., a large multinational company, owns several warehouses in some of these cities.
A salesman of ACME is planning a business trip: he will start his trip from a warehouse in a city $i$, where he will collect the products to sell, and he will end the trip in a city $j$ with $j \geq i$, where he will drop the unsold products in a warehouse. The salesman will sell the products in each of the cities encountered during the trip: when he encounters a city $k \in[i, j]$ he expects to sell goods for $s_{k}$ dollars. The salesman also knows that travelling from a generic $k$-th city to the next city $k+1$ costs $c_{k}$. This means that if he chooses $i$ and $j$ as the endpoints of his trip, his expected earnings will be of $E(i, j)=\sum_{k=i}^{j} s_{k}-\sum_{k=i}^{j-1} c_{k}$ dollars.
Design an algorithm that, given (i) the locations of the cities, (ii) the set $W$ of the (indexes of the) cities that contain ACME warehouses, (iii) the costs $c_{k}$, and (iv) the expected amount $s_{k}$ of goods sold in the generic $k$-th city, finds the best possible trip for the salesman, i.e., computes two indices $i, j \in W$ with $j \geq i$ such that $E(i, j)$ is maximized.
Input. The input consists of a set of instances, or test-cases, of the previous problem. The first line of the input contains the number $T$ of test-cases. The first line of each test-case contains the overall number of cities $n$ and the number $|W|$ of cities with a warehouse. The second line contains the $n$ values $s_{1}, \ldots, s_{n}$. The third line contains the $n-1$ values $c_{1}, \ldots, c_{n-1}$. Finally, the fourth and last line of the test-case contains $|W|$ integers corresponding to the indices of the cities in $W$, in increasing order.
Output. The output consists of $T$ lines, where the $i$-th line is the answer to the $i$-th test-case and contains $\max _{\substack{i, j \in W \\ i<j}} E(i, j)$.
Assumptions. $1 \leq T \leq 10 ; \quad 1 \leq n<2^{21} ; \quad 0<s_{i}, c_{i}<2^{10} ; \quad 2 \leq|W| \leq n$.

## Example.



Input:
1
85
37261351
2482318
24578

Output (corresponding to the trip from the 4 -th to the 7 -th city):
9
Requirements. Your algorithm should require $O(n)$ time (with reasonable hidden constants).
Notes. A reasonable implementation should not require more than 1 second for each input file.

