Algorithm Design Laboratory with Applications

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Problem: *Tunnel.*

A mountain lies between two valleys, one on its west side and the other on its east side. The people of the area decide to build a tunnel to connect the two valleys. The candidate area for excavation can be described as a rectangular $h \times \ell$ matrix M in which each entry (i, j) represents a square area. Due to the different types of rocks, different areas can be easier or harder to mine: in general, the area at coordinates (i, j) requires $h_{i,j}$ hours to clear.

The tunnel can start from any location on the east side, i.e., from any entry $(i, 1)$ with $i = 1, \ldots, h$, and can end in any location on west side, i.e., in any entry (i, ℓ) with $i = 1, \ldots, h$. Moreover, for each $j = 1 \ldots, \ell$ only one entry among $\{(1, j), \ldots, (h, j)\}$ can be excavated. Finally, the tunnel needs to be continuous, i.e., if (i, j) and $(i', j+1)$ are excavated then $|i-i'| \leq 1$.

Design an algorithm that finds the tunnel connecting the west valley to the east valley that requires the least amount of hours H to be built.

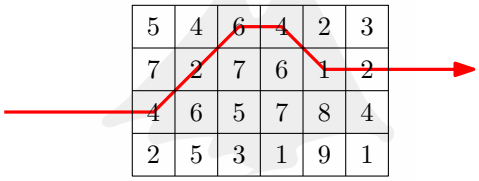
Input.

The input consists of a set of instances, or *test-cases*, of the previous problem. The first line contains the number T of test-cases. The first line of each test case contains the integers h and ℓ. The i-th of the following h lines of the test-case contains the ℓ integers $h_{i,1}, \ldots, h_{i,\ell}$.

Output.

The output consists of T lines, each corresponding to a test-case. The i-th of the lines contains the integer H corresponding to the minimum amount of hours needed to build a tunnel from the west valley to the east valley.

Example.



Input (corresponding to the above example):

```
1
4 6
5 4 6 4 2 3
7 2 7 6 1 2
4 6 5 7 8 4
2 5 3 1 9 1
```

Output:

```
19
```

Assumptions. $1 \leq T < 10$; $1 \leq h < 2^{10}$; $1 \leq \ell < 2^{13}$; $\forall i = 1, \ldots, h, \forall j = 1, \ldots, \ell$, $1 \leq h_{i,j} \leq 2^{14}$.

Requirements. Your algorithm must have an asymptotic time complexity of $O(h \cdot \ell)$ (with reasonable hidden constants).

Notes. A reasonable implementation should not require more than 1 second for each input file.