String Matching

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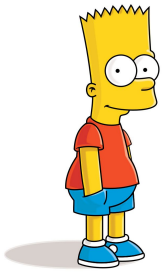
Problem: Given an alphabet Σ, a *text* $T \in \Sigma^*$ and a *pattern* $P \in \Sigma^*$, find some occurrence/all occurrences of P in T.



$$\Sigma = \{\texttt{A}, \texttt{B}, \ldots, \texttt{Z}, \texttt{a}, \texttt{b}, \ldots, \texttt{z}, \texttt{␣}\}$$

$$T = \texttt{Bart␣played␣darts␣at␣the␣party}$$

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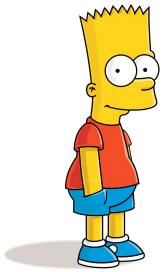

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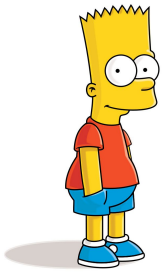

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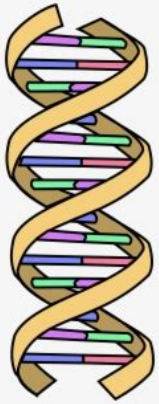

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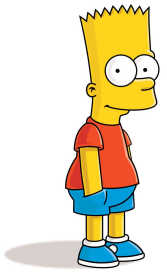

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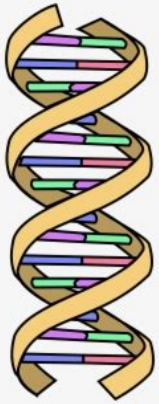

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Tries

Tries (Prounounced as "try")

Data structure to store a dynamic collection of k strings over an alphabet Σ

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- **Insert**(T): add T to the collection of strings
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Obs: A string comparison requires time $O(\text{string length})$.
Binary searching requires time $O(\text{max string length} \cdot \log k)$

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We will only focus on the static case

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Pretend that each string ends with a special "end marker" symbol $

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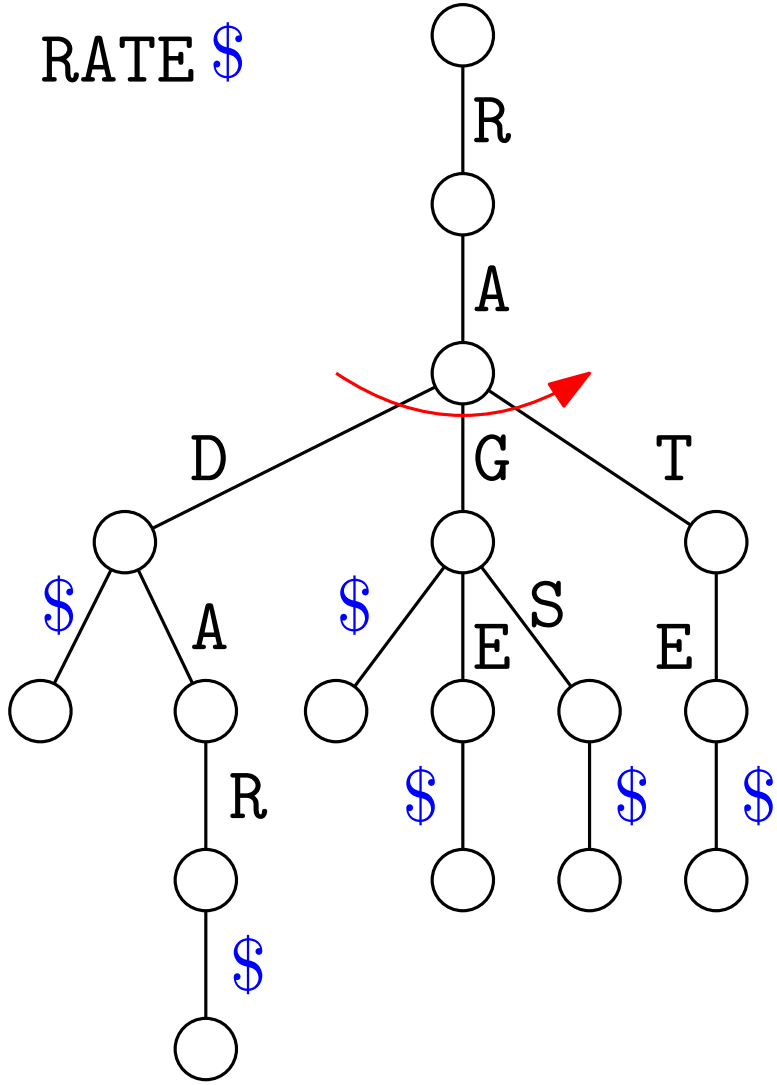

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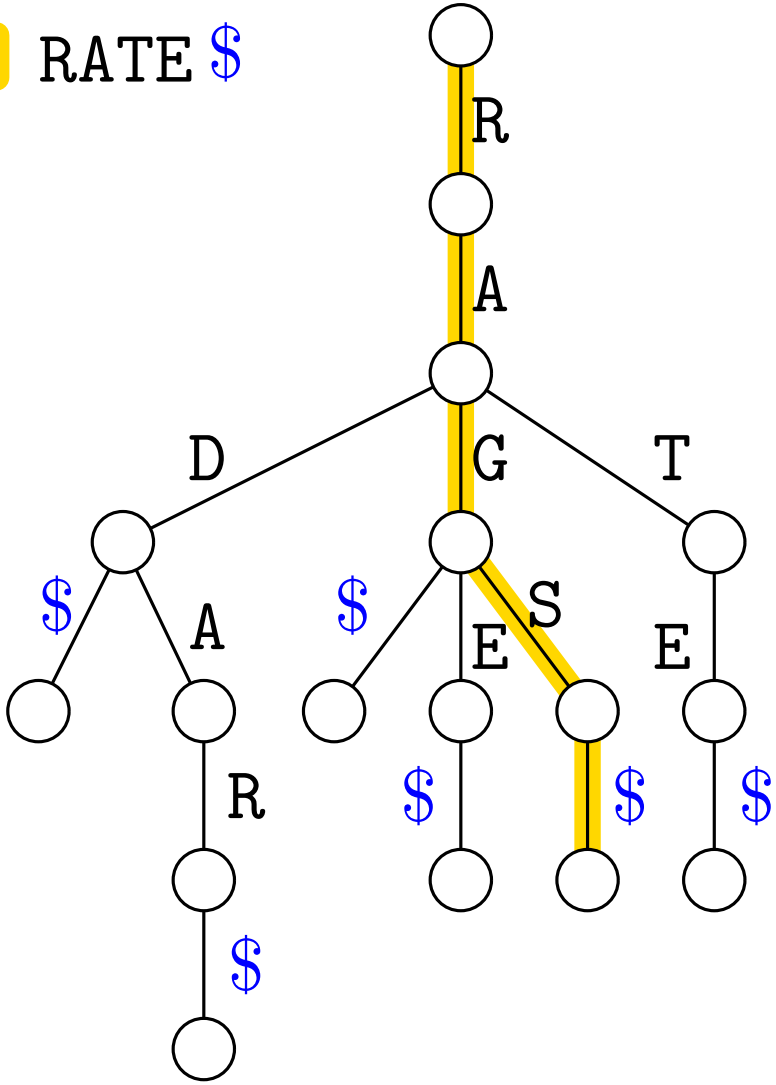

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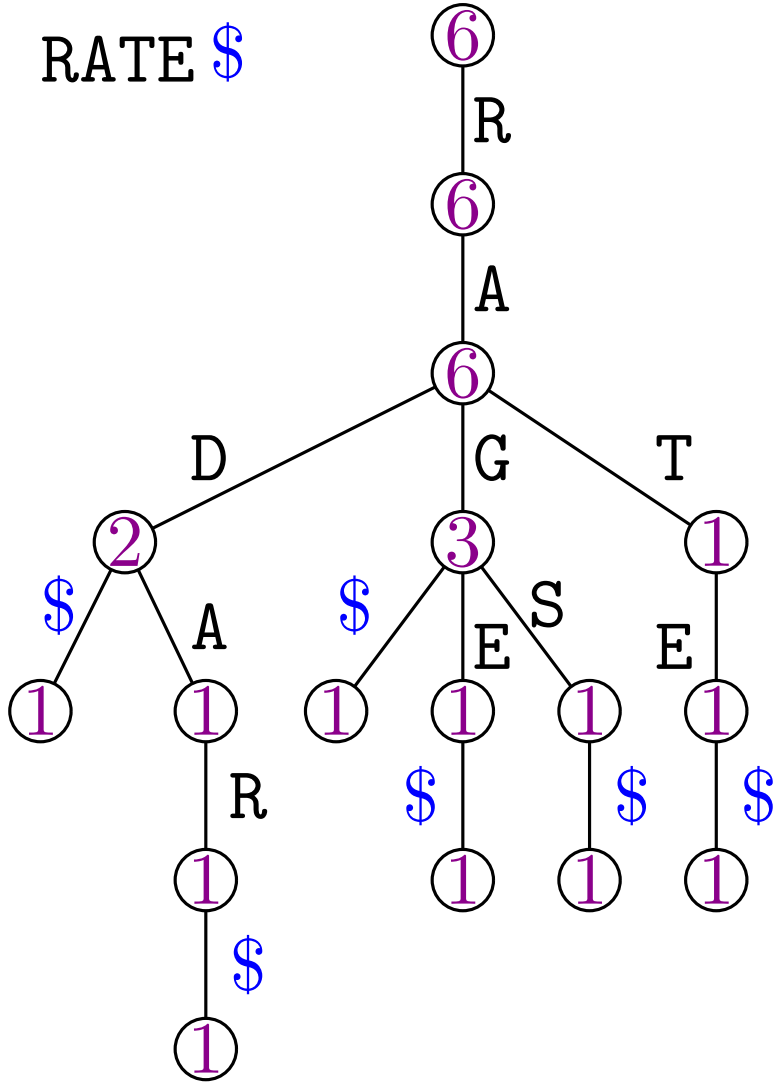

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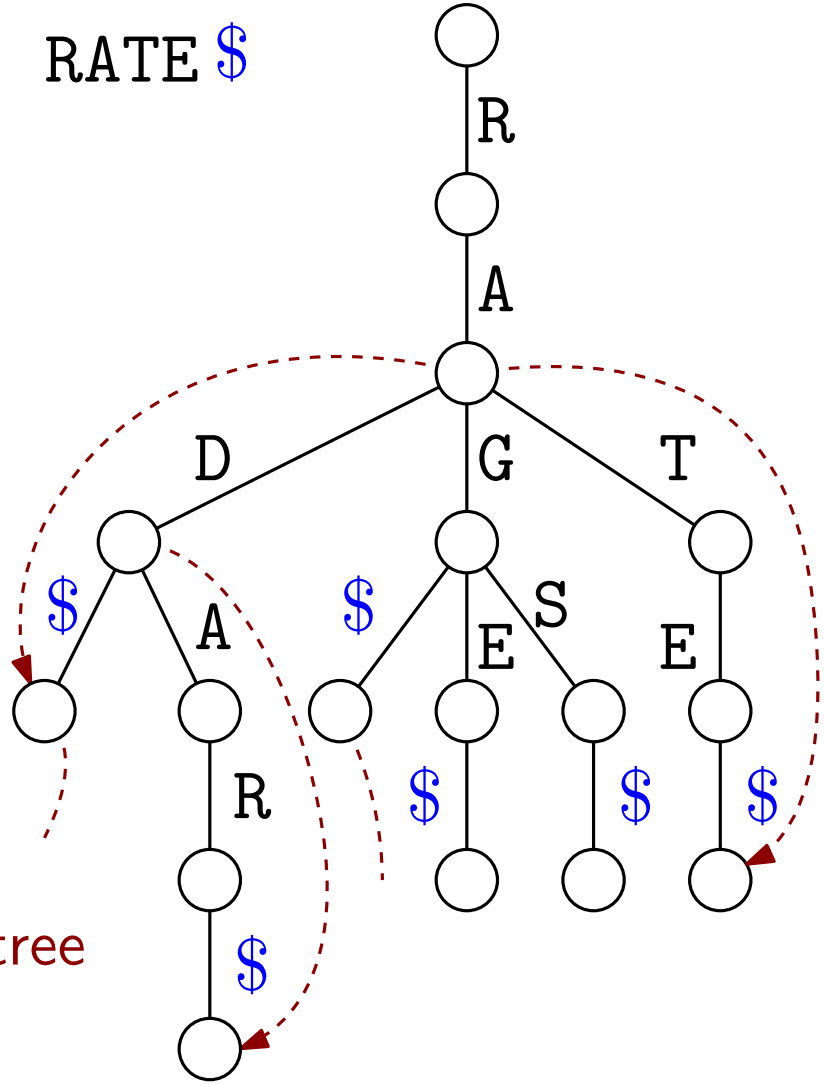

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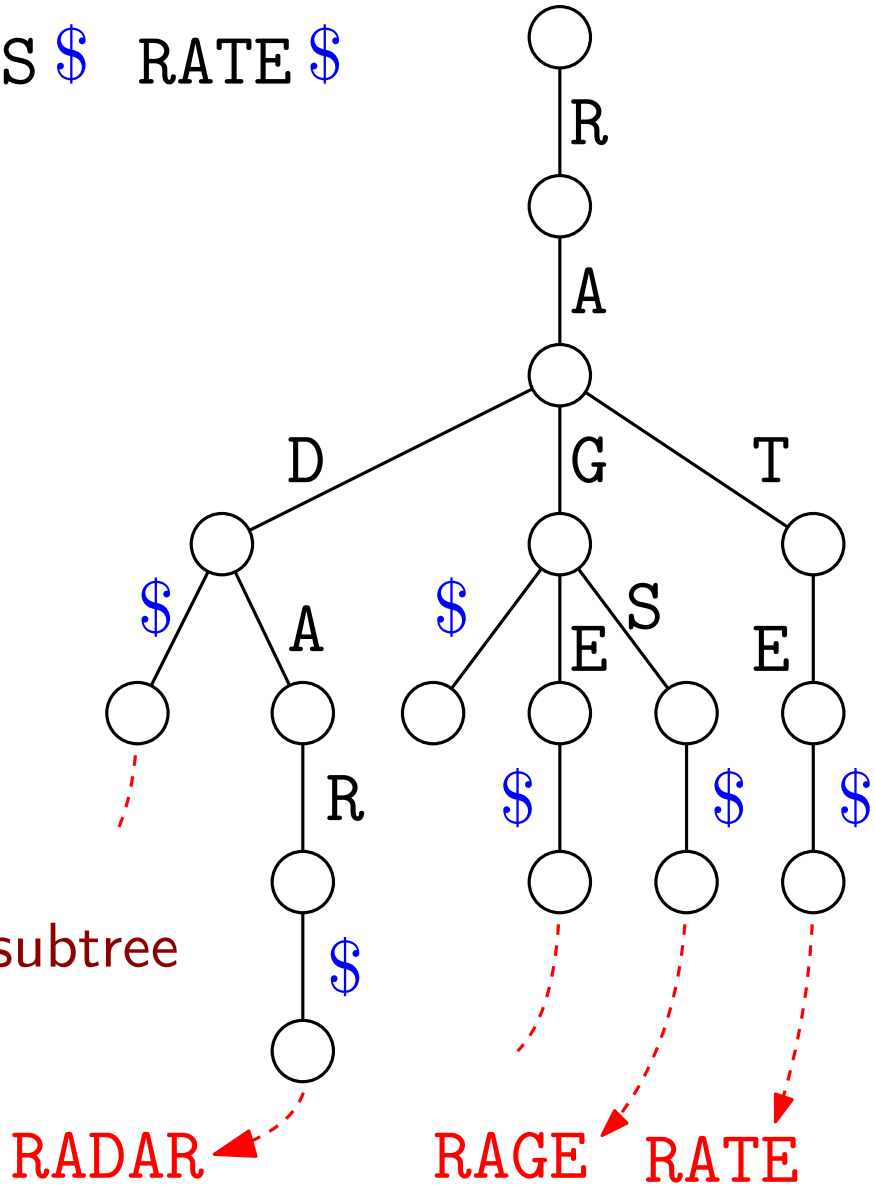

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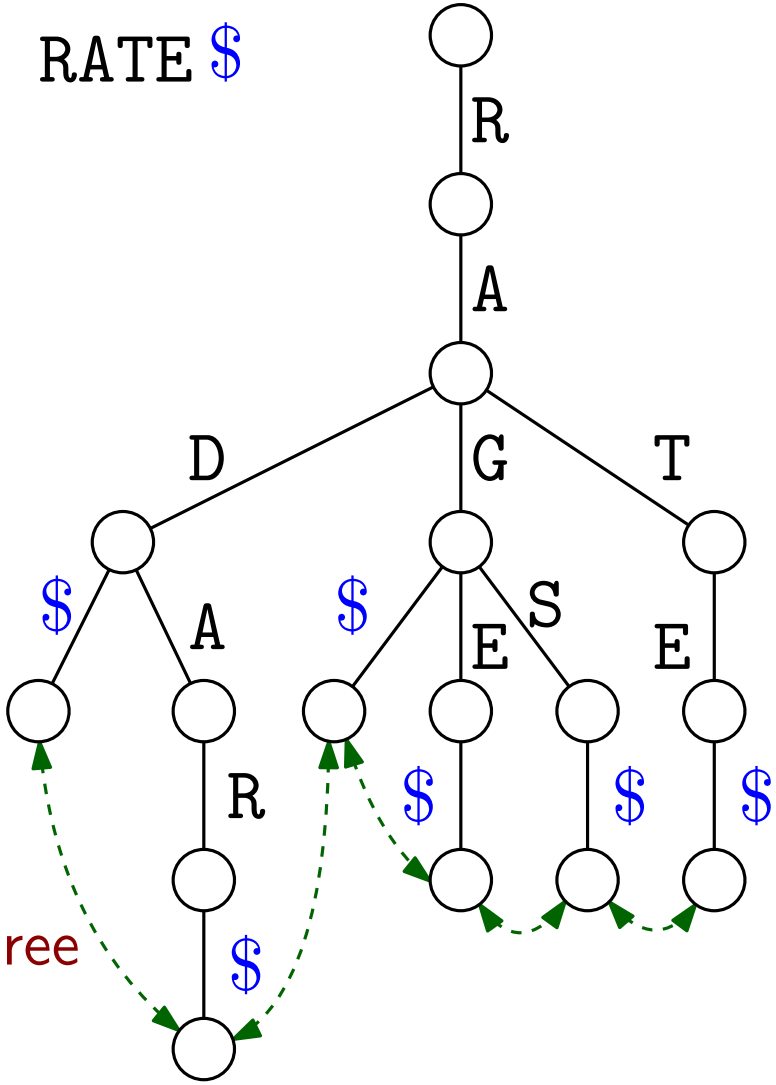
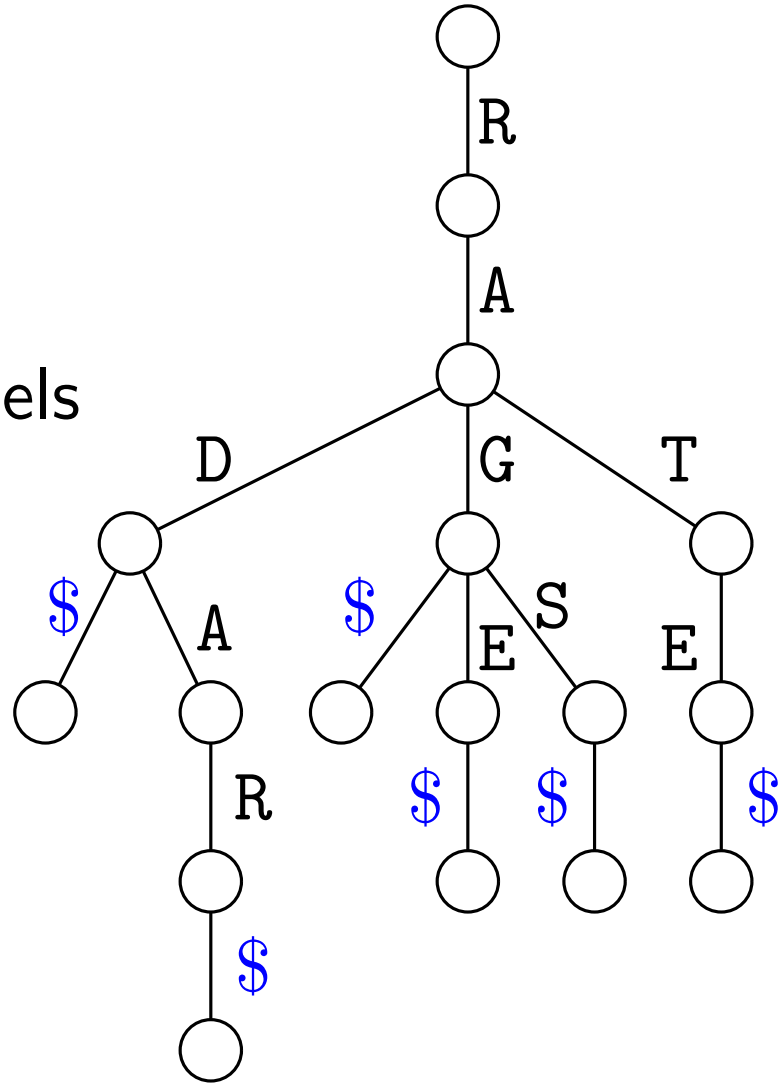
— Leaves arranged in a (doubly) linked list



Tries: Find (Sketch)

Find(P):

- Walk down the tree matching the characters in $P\$$ with the edge labels



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$P = \texttt{RADAR}$

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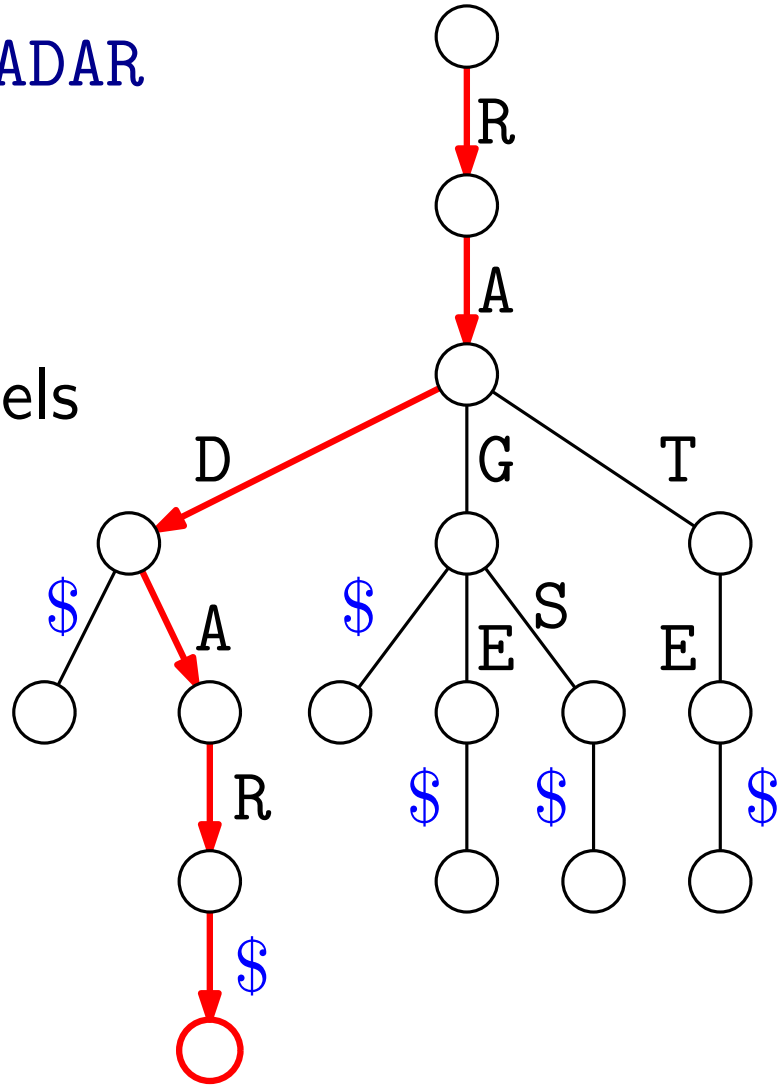

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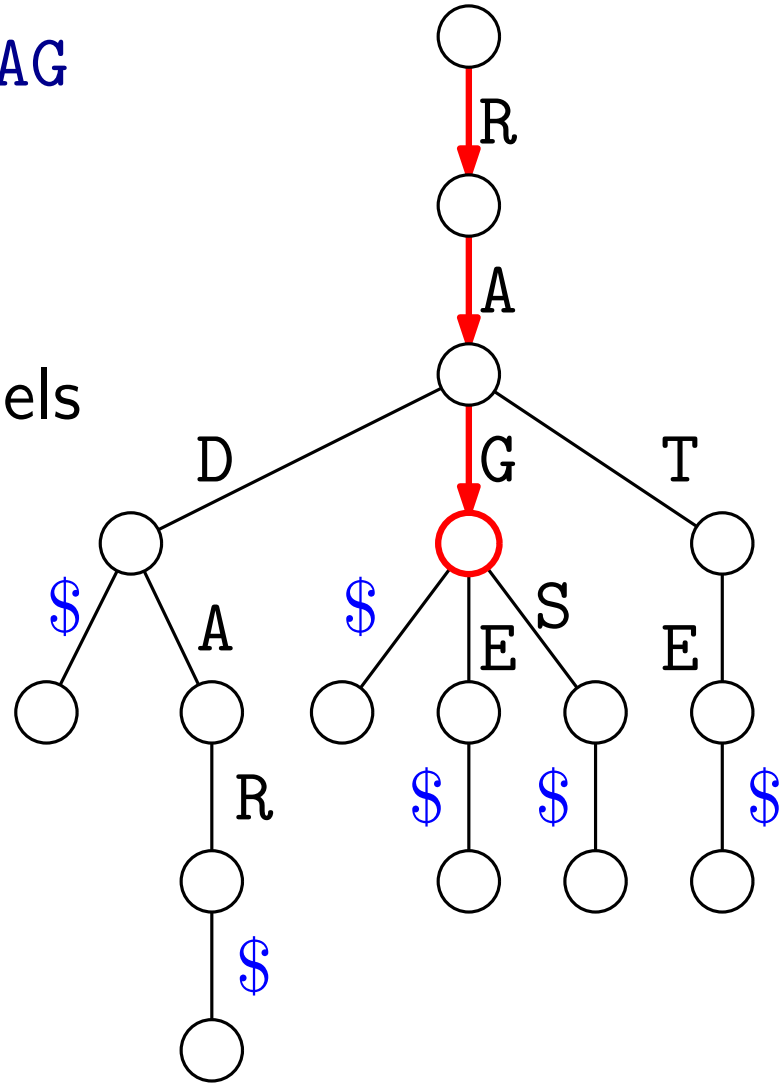
To count the number of strings that start with P:

- Find the node corresponding to P



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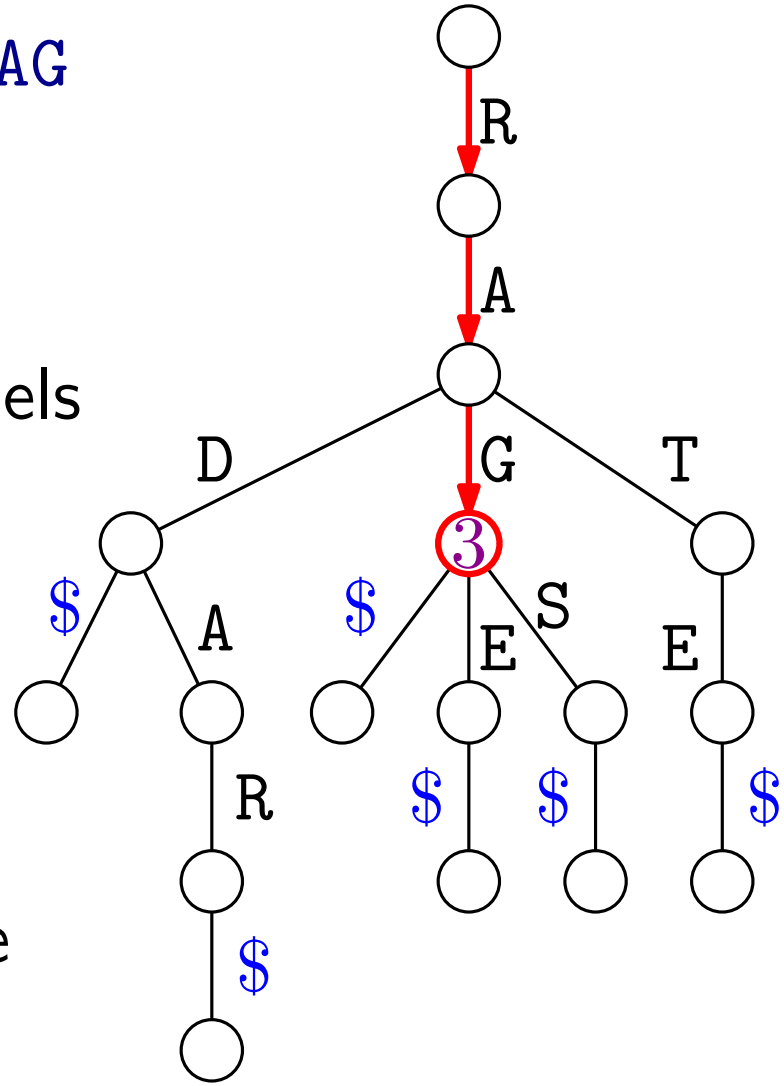

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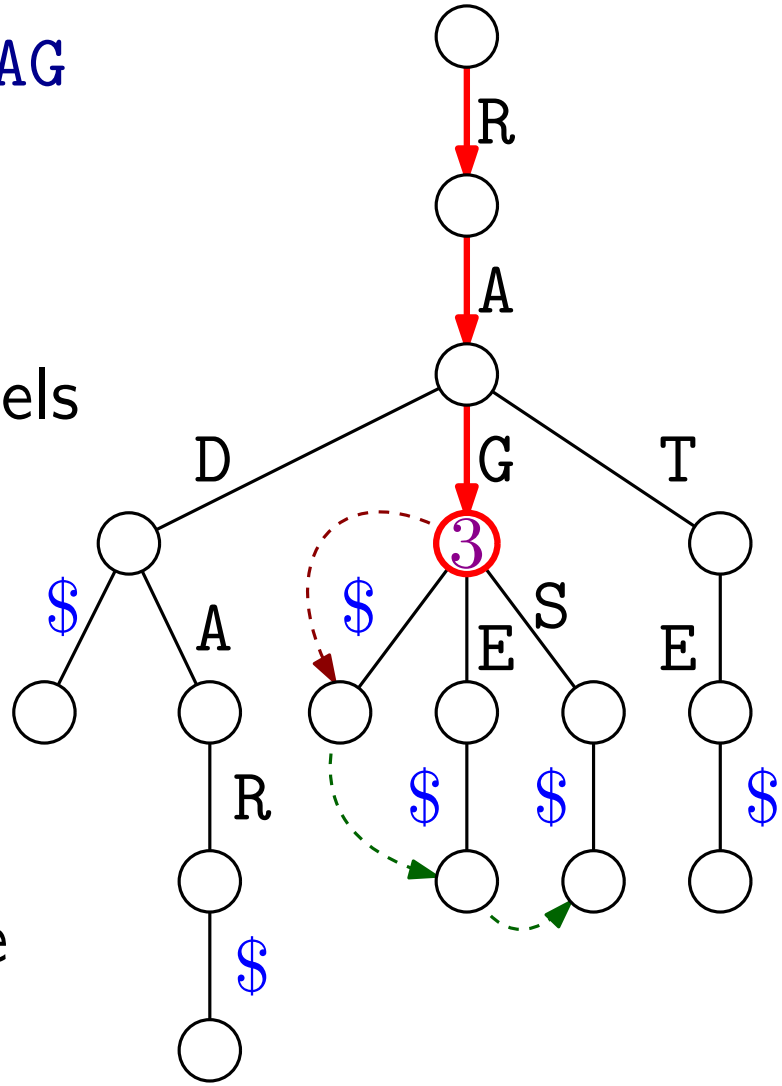

To count the number of strings that start with P:

- Find the node corresponding to P
- Return the number of leaves in the subtree (stored in the node)
- The actual matches can be listed in $O(1)$ additional time per match by following pointers

Tries: Predecessor Queries (Sketch)

$T\$ = T_1 T_2 T_3 \ldots$

$T = \texttt{RAG}$

Predecessor(T):

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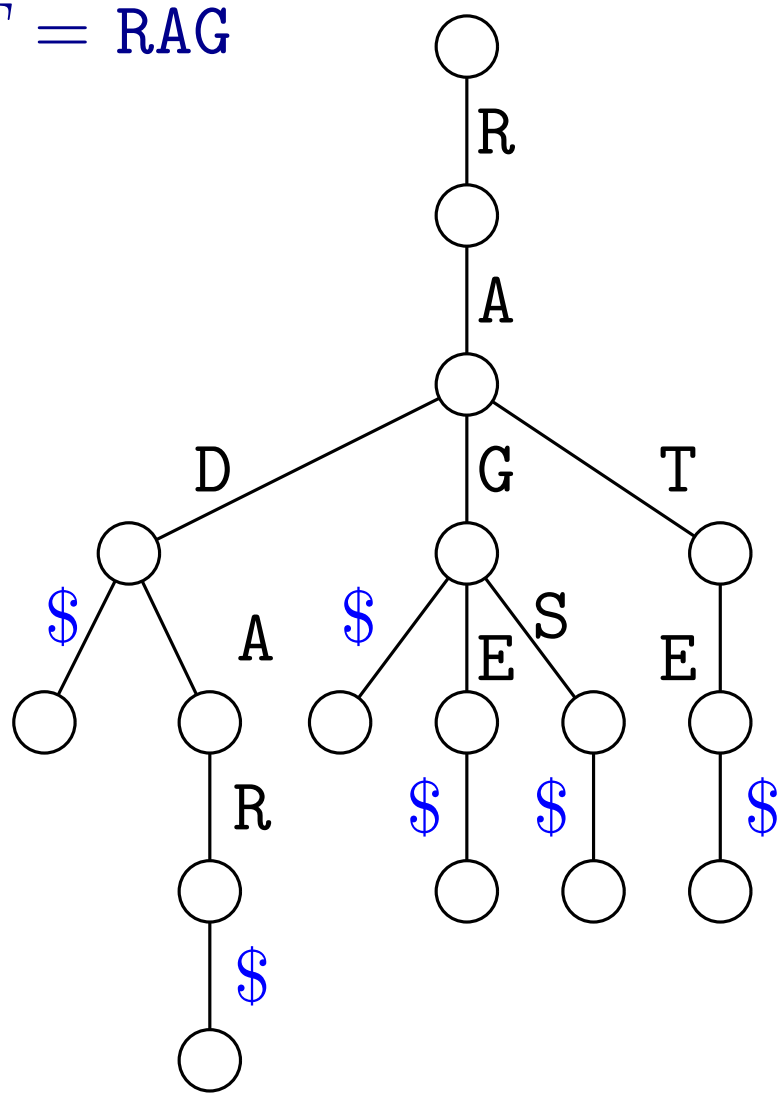

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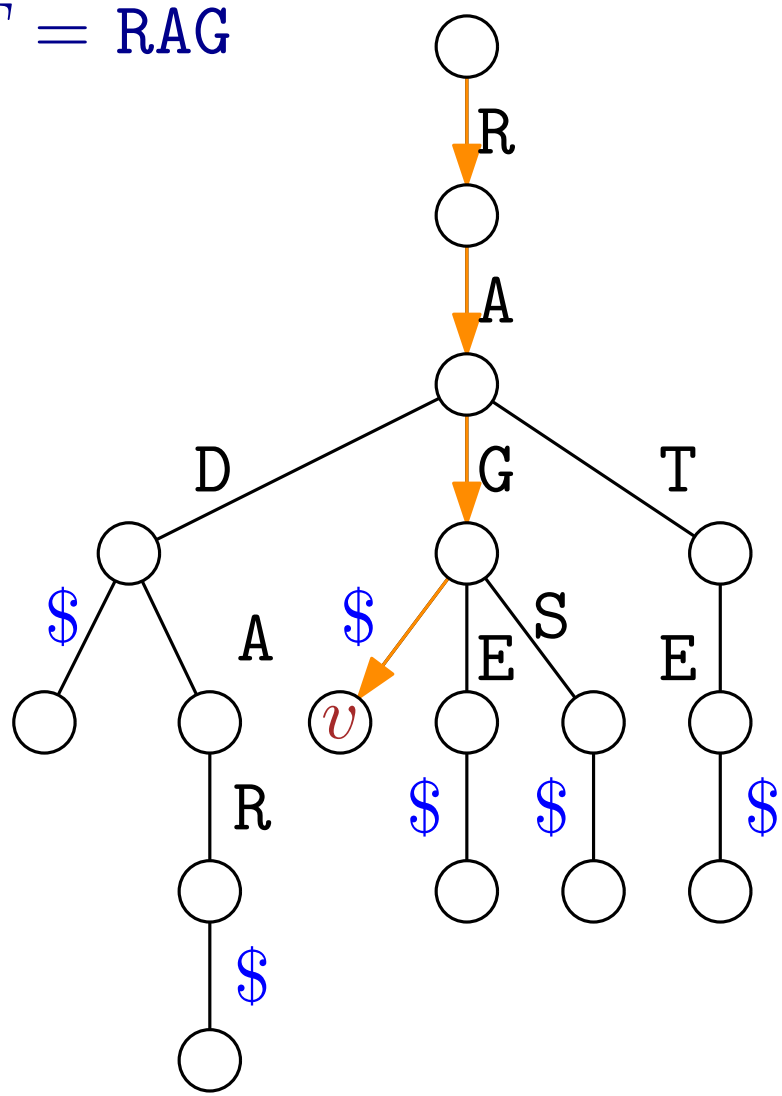

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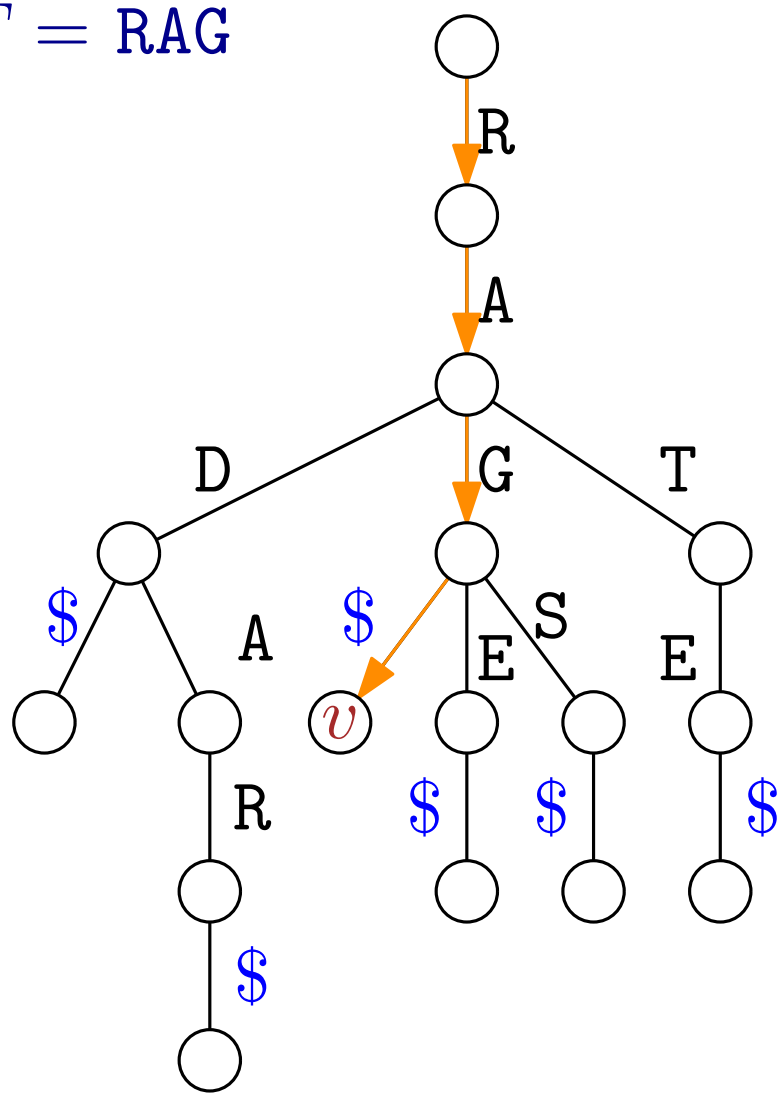

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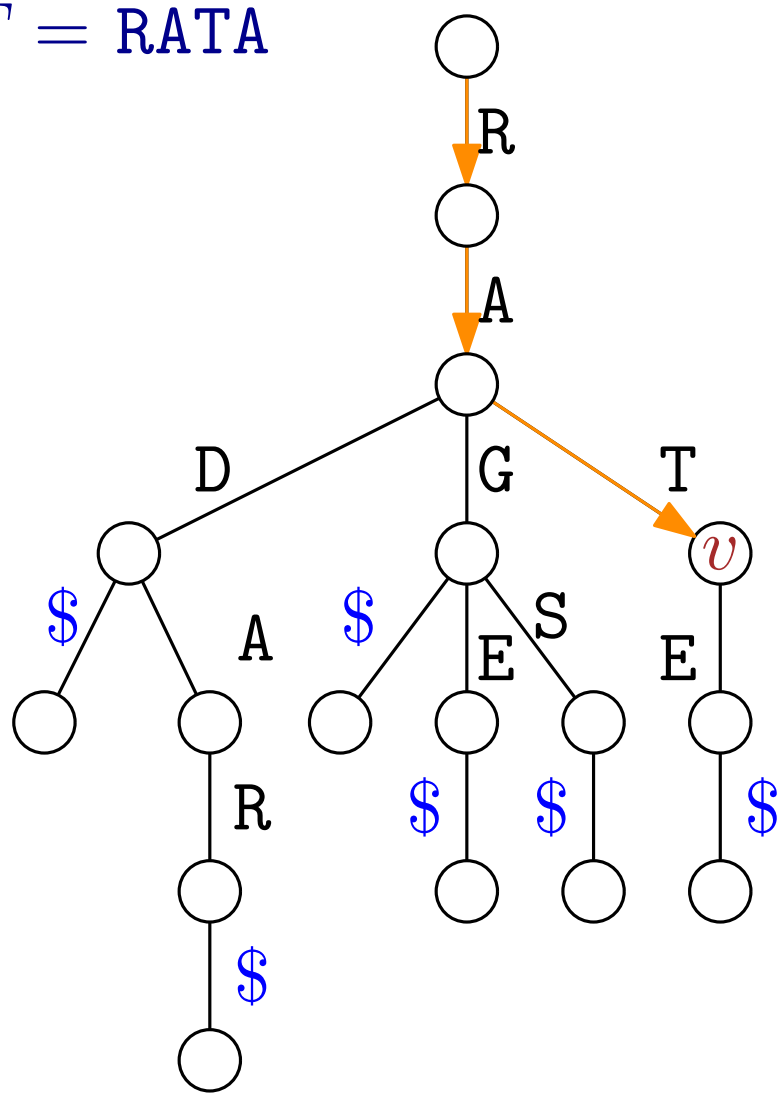

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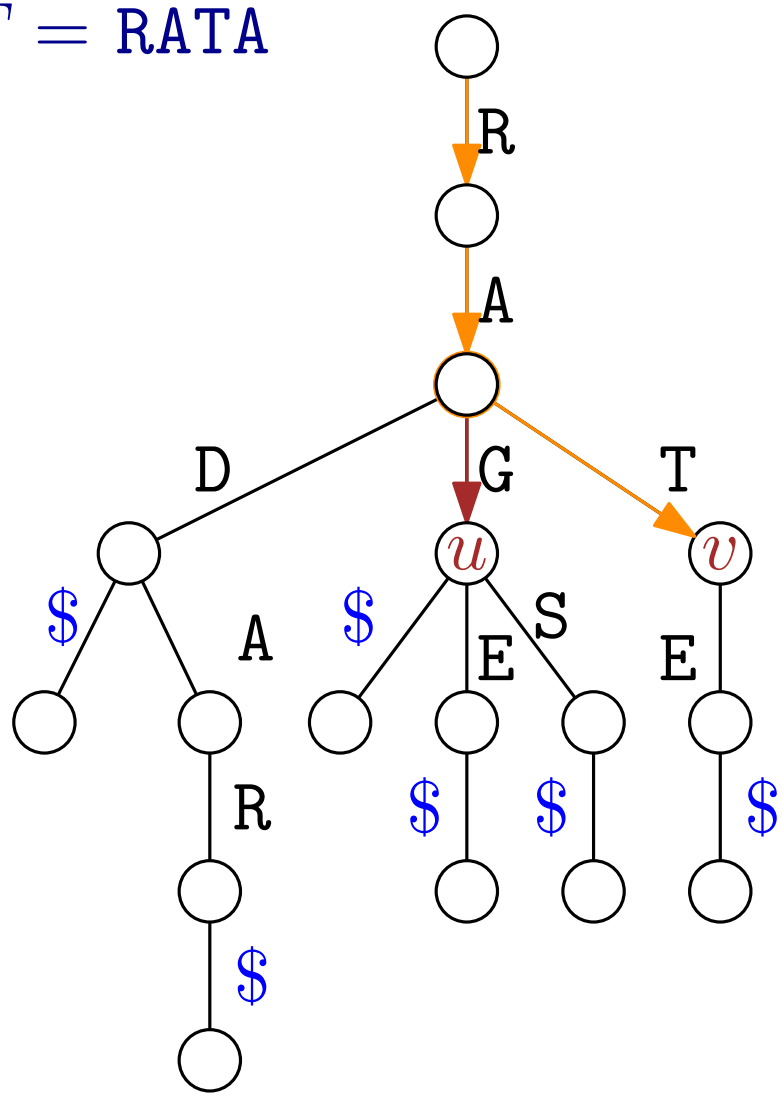

The strict predecessor of $\sigma \in \Sigma$ w.r.t. a node v, if it exists, is the child u of v such that (v, u) has the largest label that is smaller than σ

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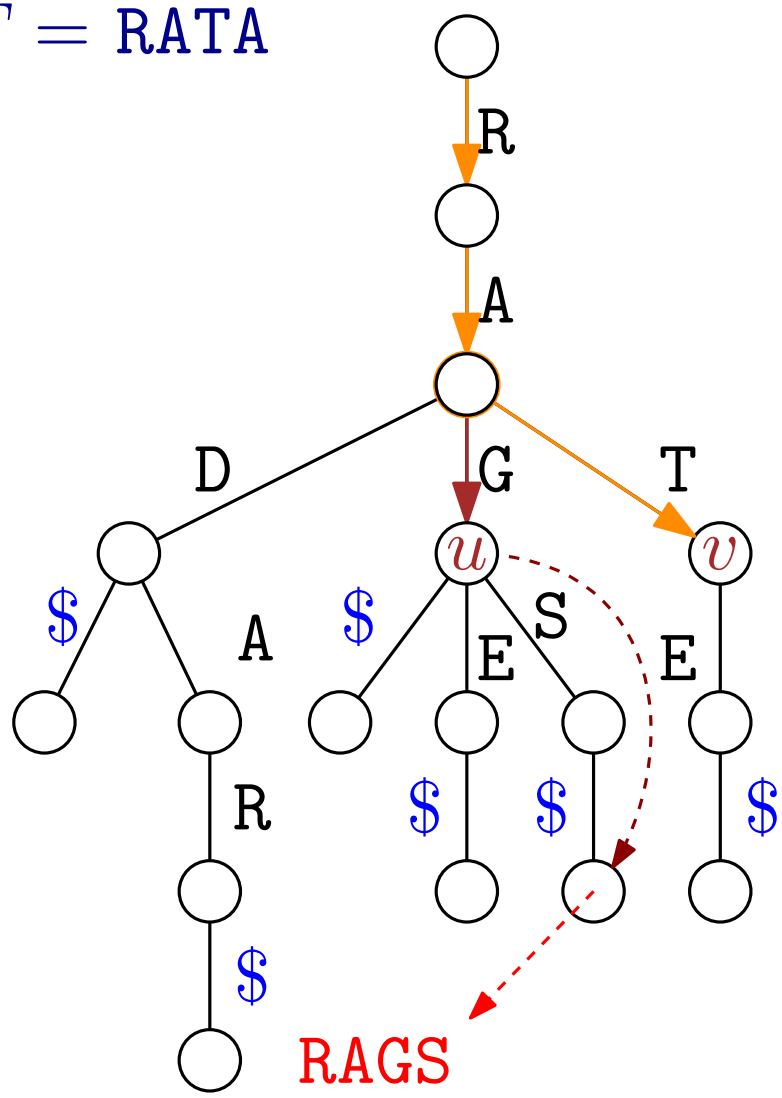

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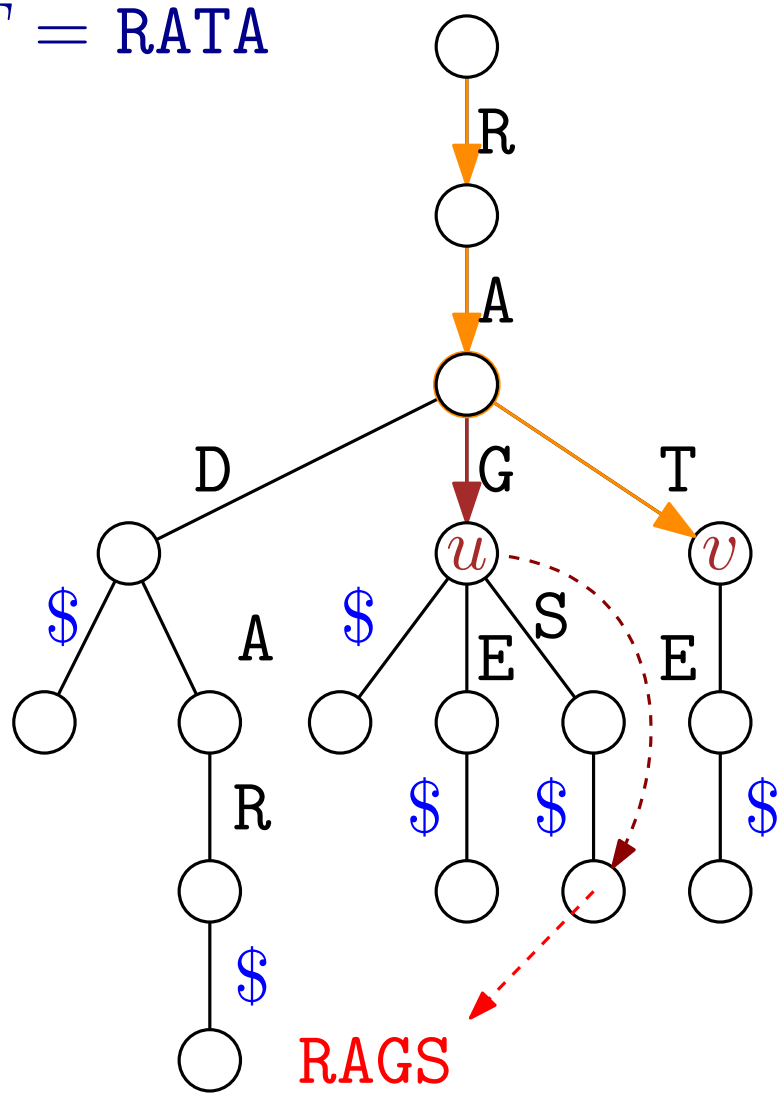

Time?

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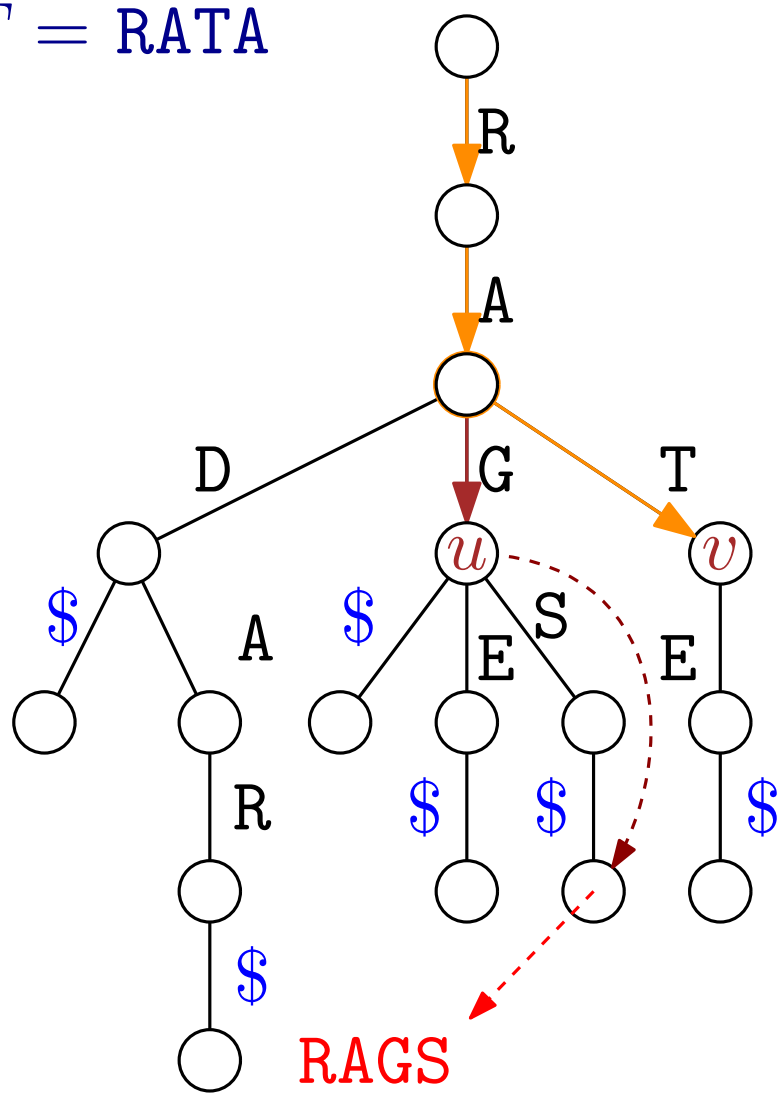

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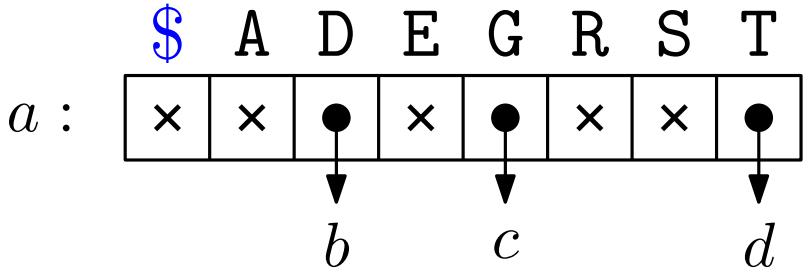
Time? Depends on how the tree is stored

Representing Tries

Array (dense)



$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

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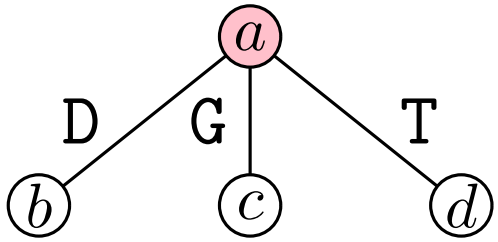

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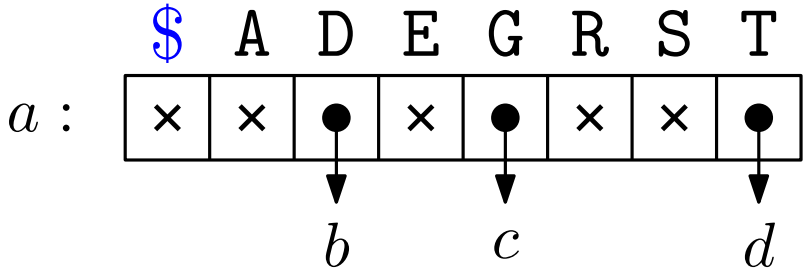

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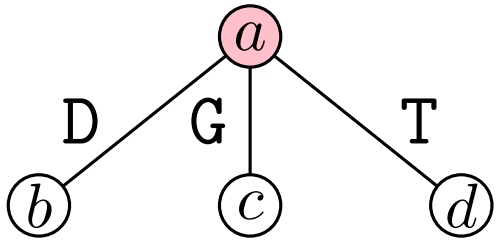
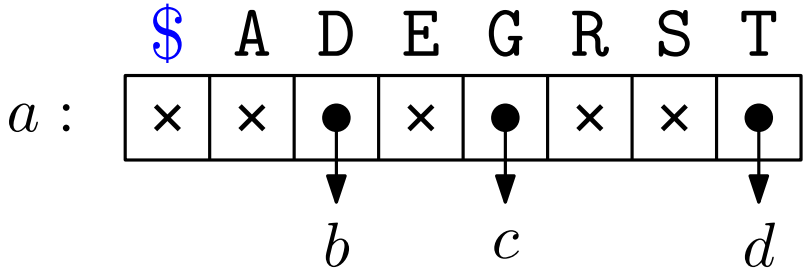


Space: $O(|\Sigma|)$

Time to find a symbol's edge: $O(1)$

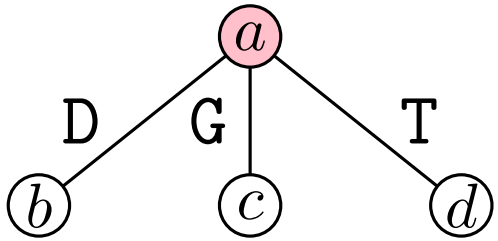
Representing Tries

Array (dense)



$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

$\Sigma = \{\texttt{A}, \texttt{D}, \texttt{E}, \texttt{G}, \texttt{R}, \texttt{S}, \texttt{T}\}$



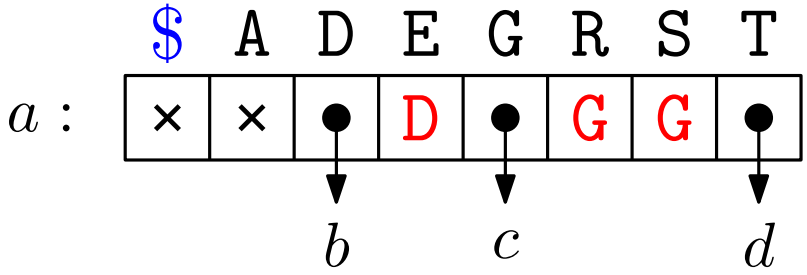
Space: $O(|\Sigma|)$

Time to find a symbol's edge: $O(1)$

Time to find predecessor: $O(|\Sigma|)$

Representing Tries

Array (dense)



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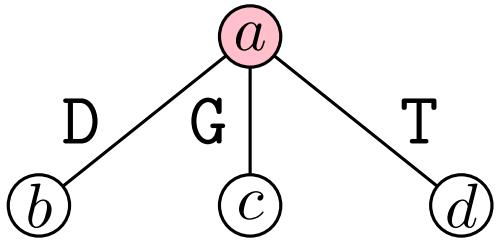

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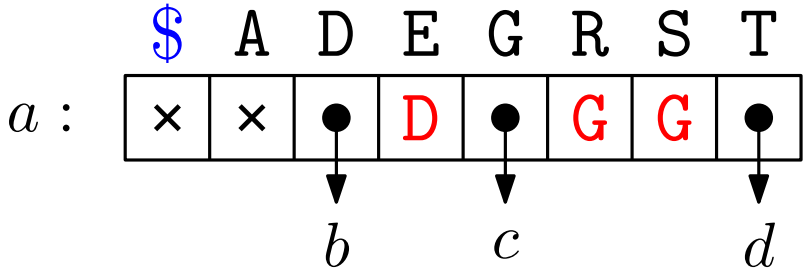
Time to find predecessor: $O(|\Sigma|)$

Representing Tries

Array (dense)



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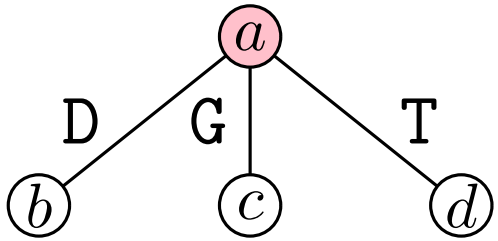


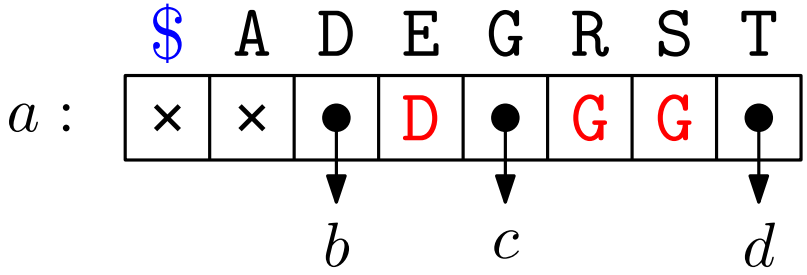
Space: $O(|\Sigma|)$

Time to find a symbol's edge: $O(1)$

Time to find predecessor: ~~$O(|\Sigma|)$~~ $O(1)$

Representing Tries

Array (dense)



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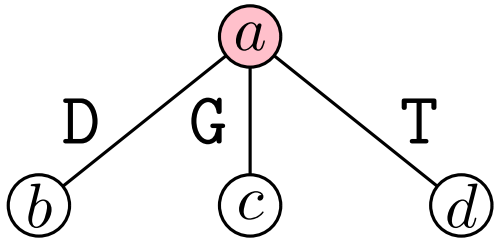


Space: $O(|\Sigma|)$

Time to find a symbol's edge: $O(1)$

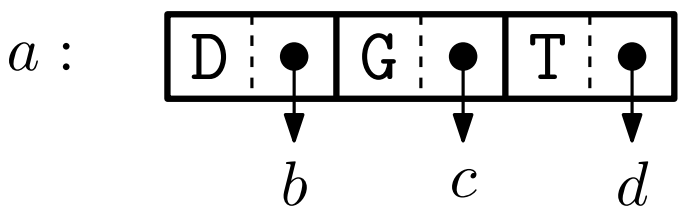
Time to find predecessor: ~~$O(|\Sigma|)$~~ $O(1)$

Overall space: $O(|\Sigma| \cdot n)$

Overall time: $O(|P|)$

Representing Tries

Array (sparse)



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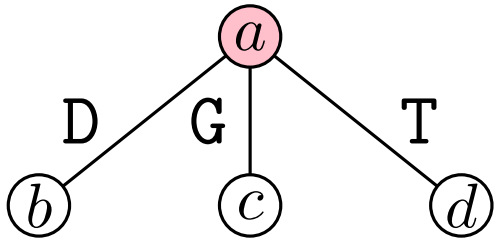
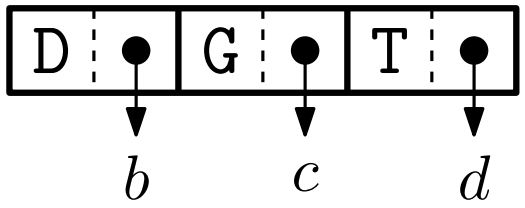


Representing Tries

Array (sparse)

a :



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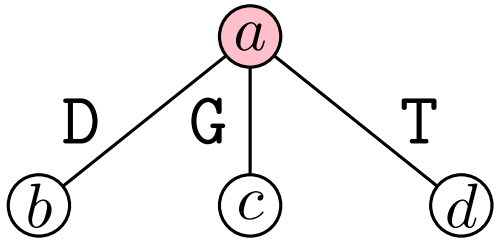
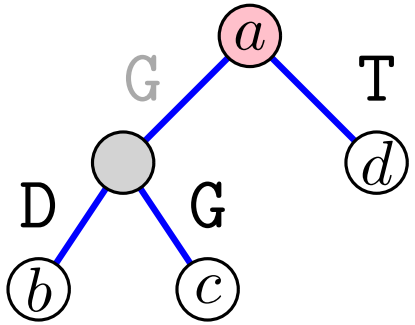


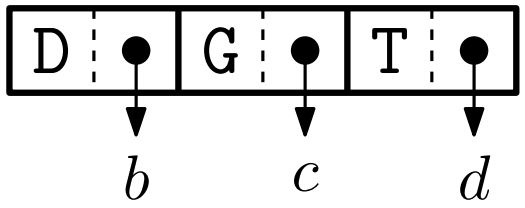
Balanced Binary Search Tree



Representing Tries

Array (sparse)

a :



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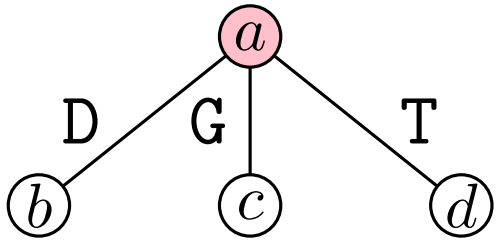
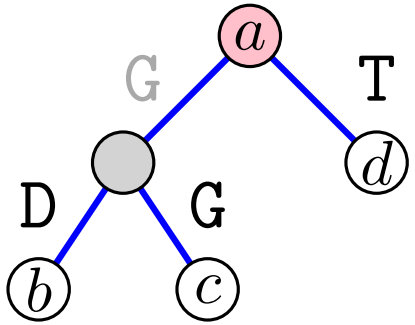


Balanced Binary Search Tree



Space: $O(\#\text{children})$

Representing Tries

Array (sparse)

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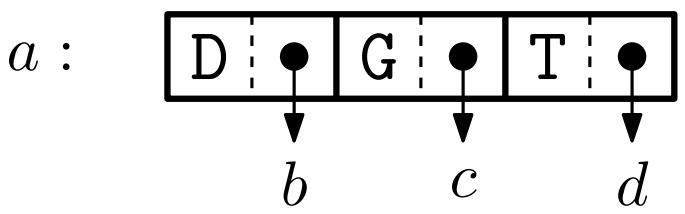
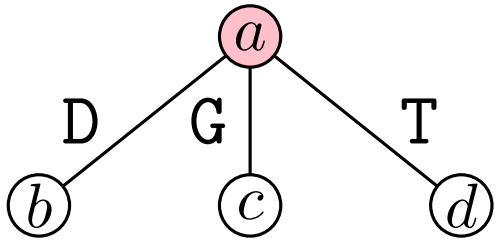
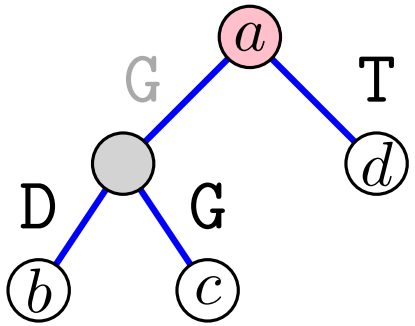




Balanced Binary Search Tree

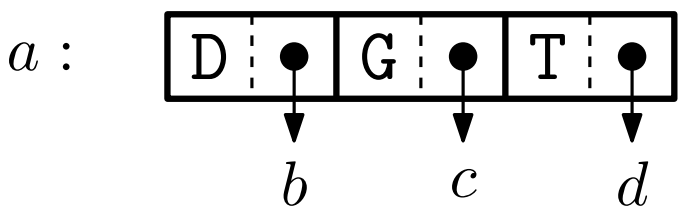


Space: $O(\#\text{children})$

Time to find a symbol's edge/predecessor:
$O(\log \#\text{children}) = O(\log |\Sigma|)$

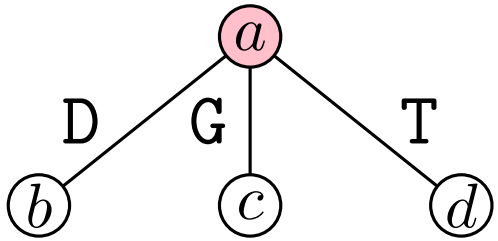
Representing Tries

Array (sparse)

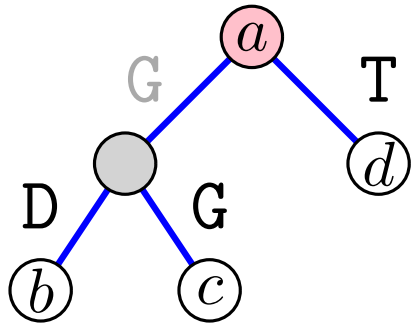


$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

$\Sigma = \{\mathtt{A}, \mathtt{D}, \mathtt{E}, \mathtt{G}, \mathtt{R}, \mathtt{S}, \mathtt{T}\}$



Balanced Binary Search Tree



Overall space: $O(n)$

Overall time: $O(|P| \log |\Sigma|)$

Space: $O(\#\text{children})$

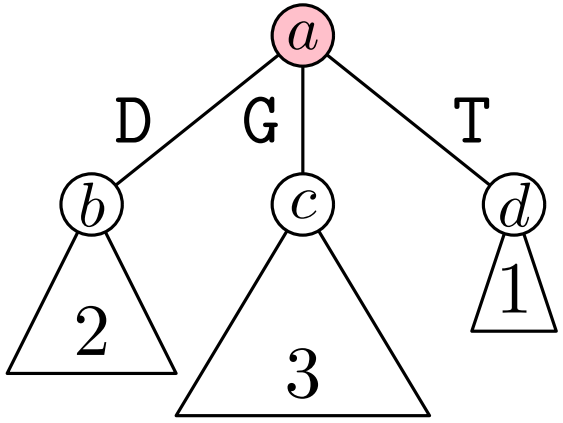
Time to find a symbol's edge/predecessor:
$O(\log \#\text{children}) = O(\log |\Sigma|)$

Representing Tries

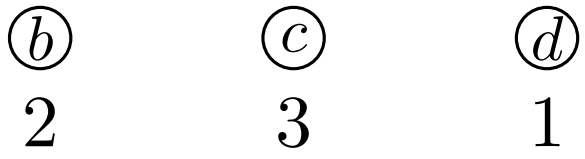
Weight-Balanced BSTs

$n = \#\text{nodes} = O\left(\sum_i |T_i|\right)$

Each vertex of the trie has a weight equal to the number of leaves in its subtree



Recursively construct a binary search tree by splitting the children in the trie so that the sum of their weights is as balanced as possible



Representing Tries

Weight-Balanced BSTs

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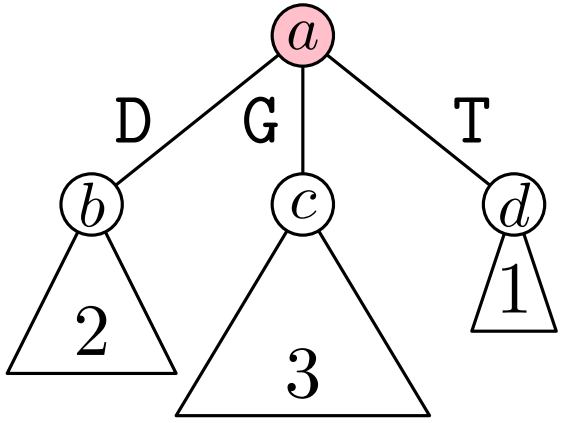

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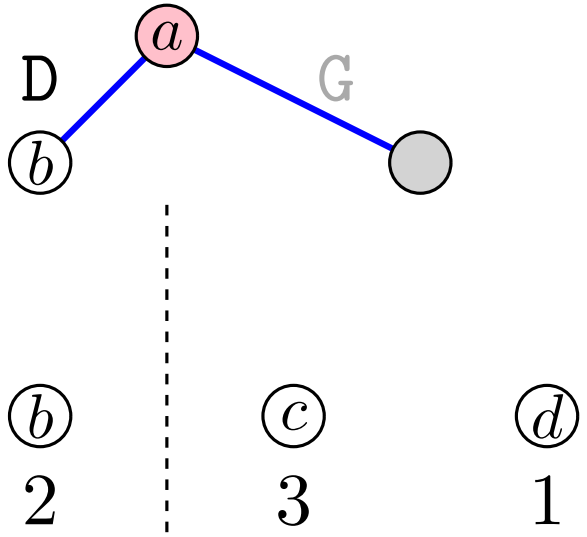

Representing Tries

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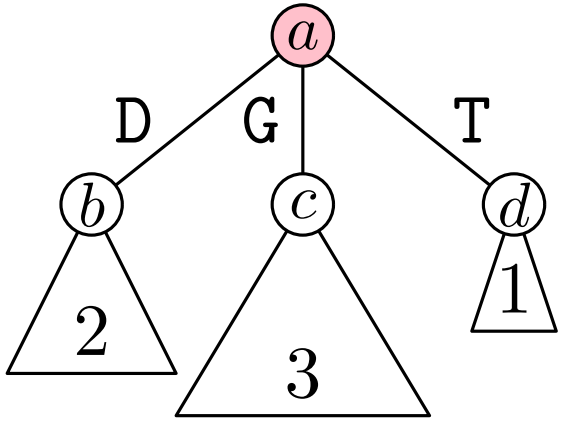

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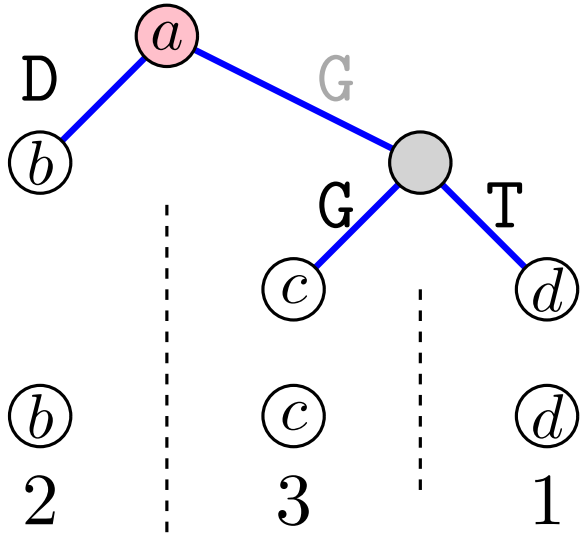

Representing Tries

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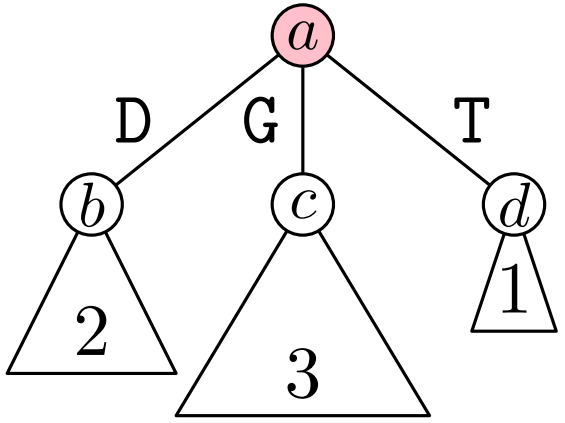

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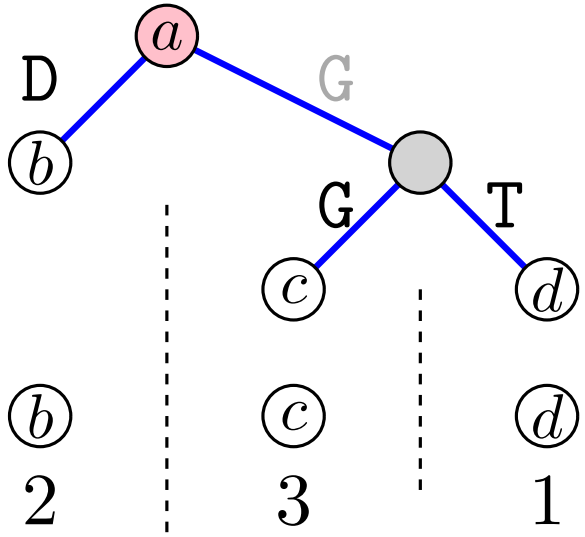


Space: $O(\#\text{children})$

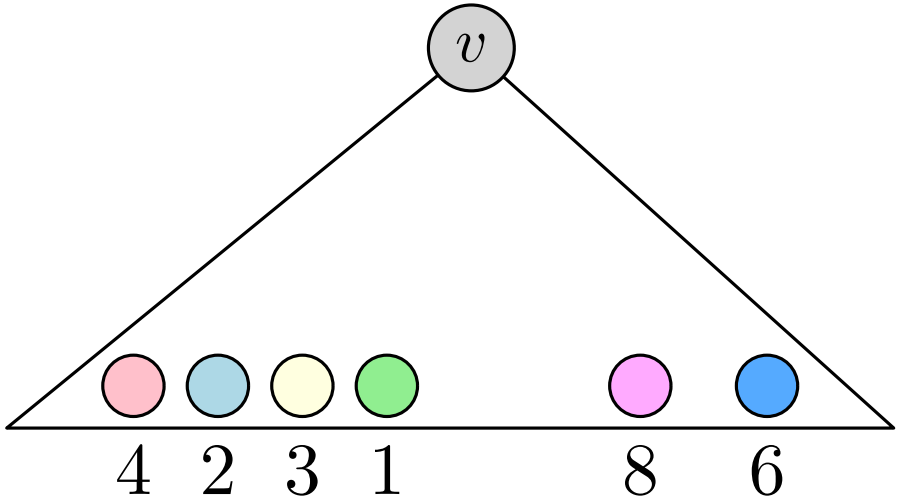
Overall space: $O(n)$

Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}w(v)$ or are leaves.

Imagine the leaves in the subtree of v as consecutive segments with lengths equal to their weights



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Weight-Balanced BSTs

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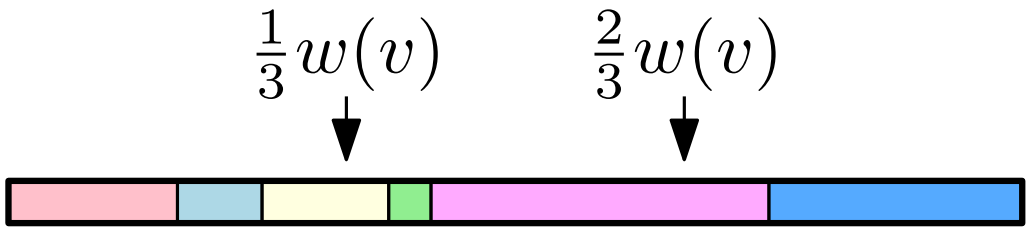
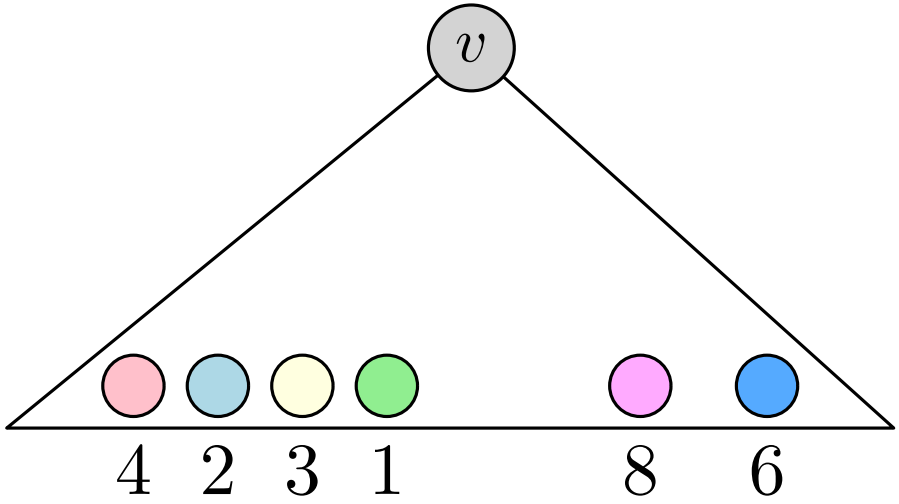
If the interval $[\frac{1}{3}w(v), \frac{2}{3}w(v)]$ contains more than one segment:





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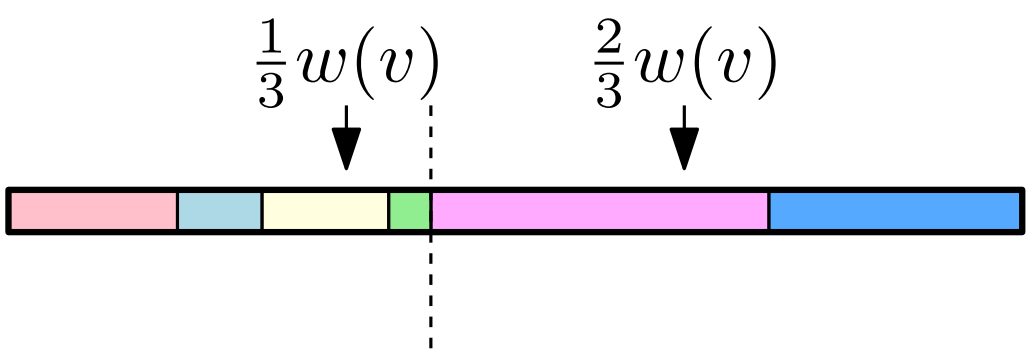

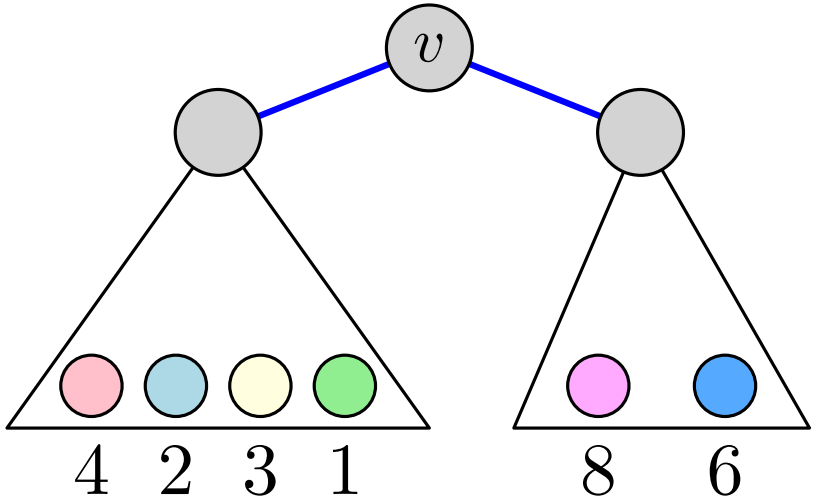

Representing Tries

Weight-Balanced BSTs

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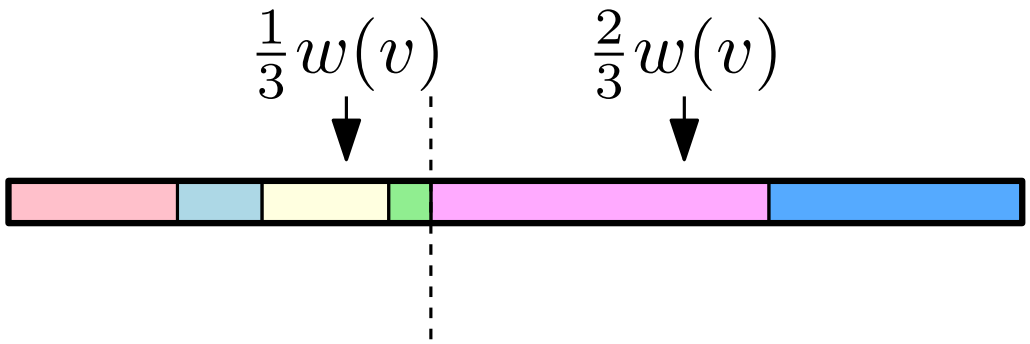
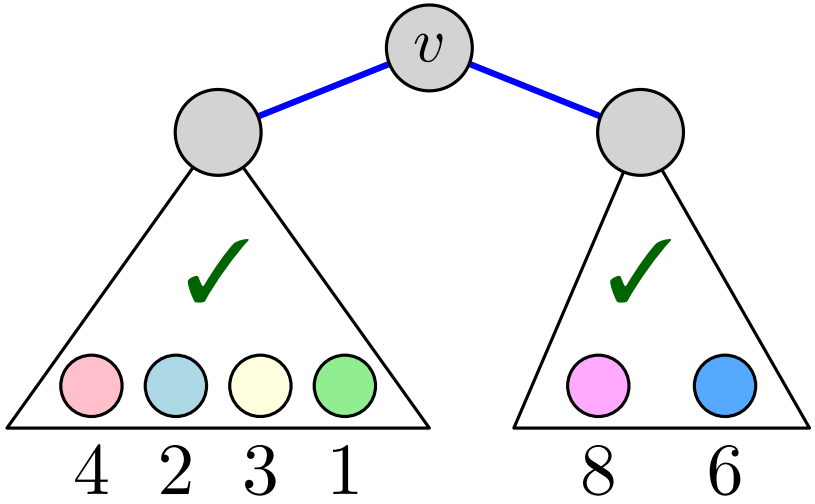




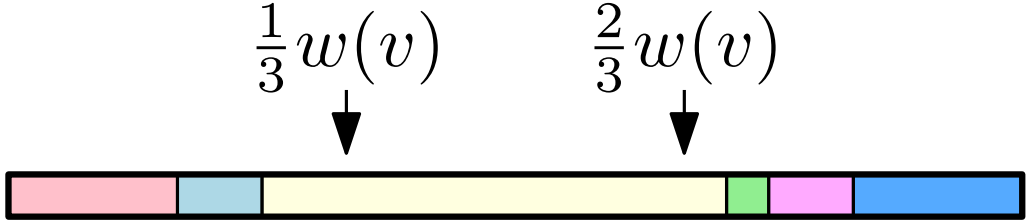
- The weight of each children of v is at most $\frac{2}{3}w(v)$

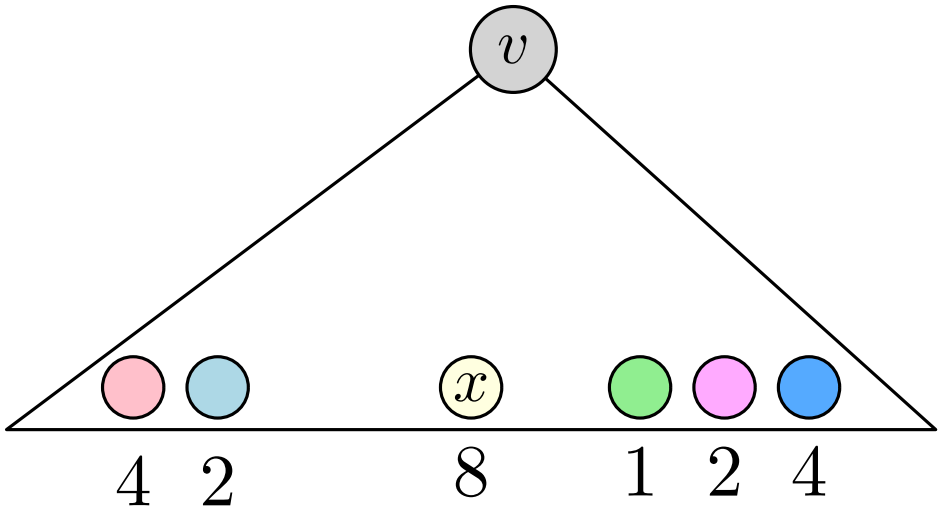
Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}w(v)$ or are leaves.

If the interval $[\frac{1}{3}w(v), \frac{2}{3}w(v)]$ contains a single segment, let x be the corresponding leaf





Representing Tries

Weight-Balanced BSTs

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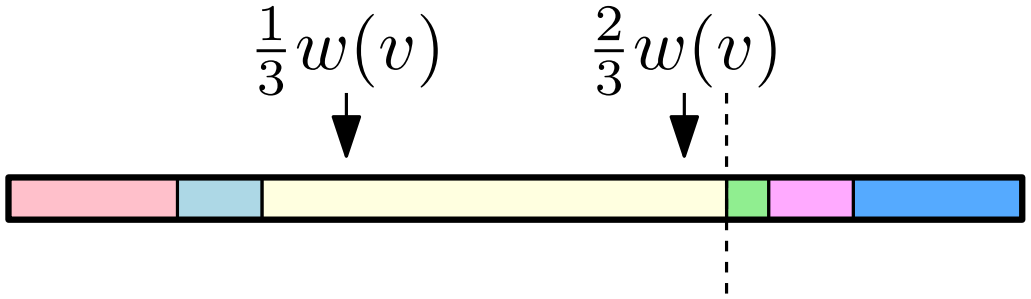
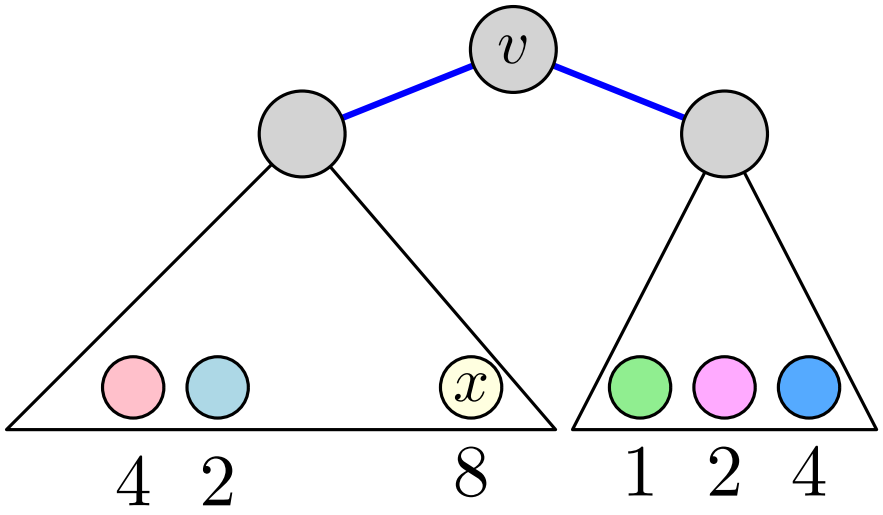


- v splits the segments immediately before/after x.



Representing Tries

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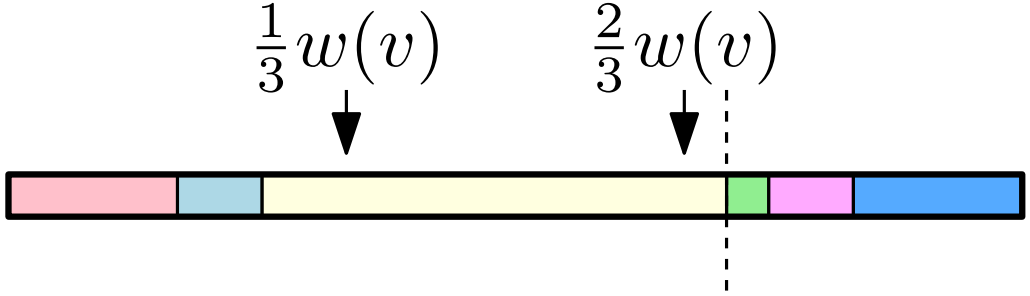

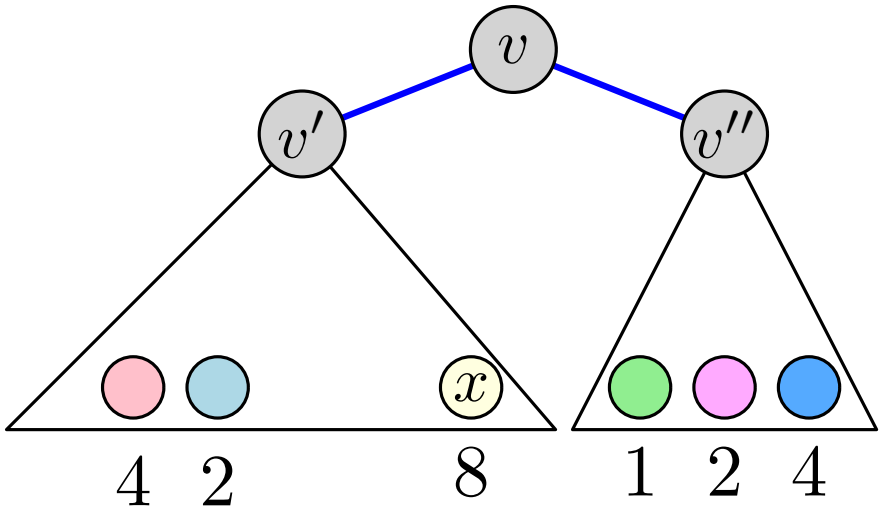

- v splits the segments immediately before/after x.
- Let v' be the child of v that contains x and let v'' be the other child

Representing Tries

Weight-Balanced BSTs

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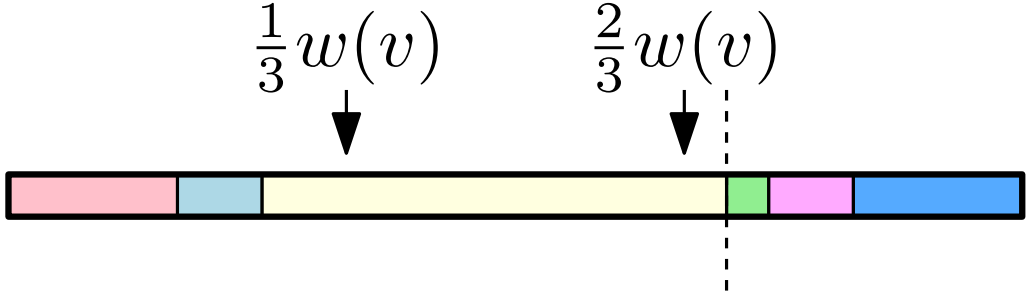

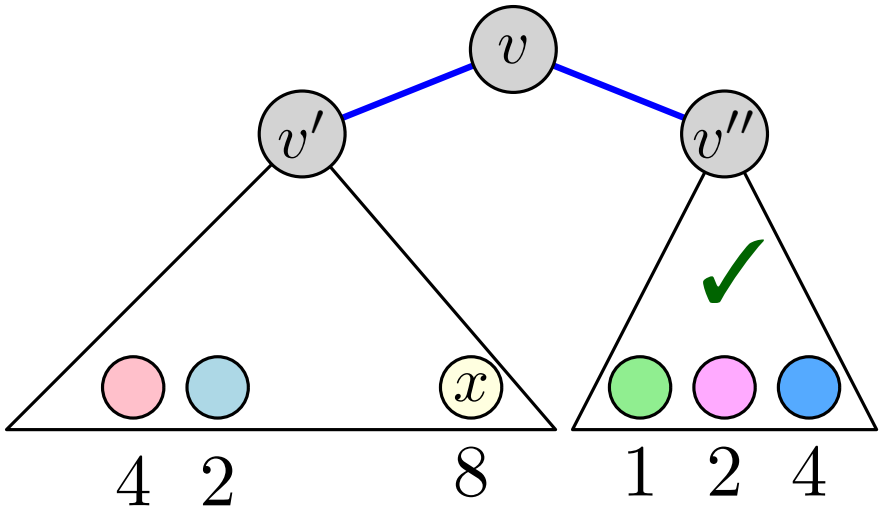

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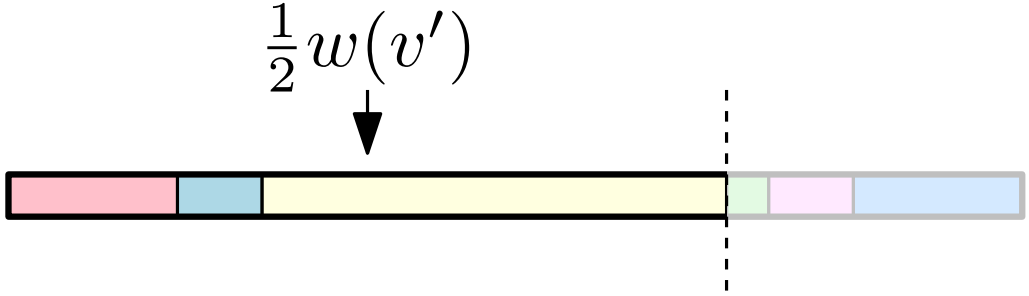
- Let v' be the child of v that contains x and let v'' be the other child
- $w(v'') \leq \frac{1}{3}w(v)$.

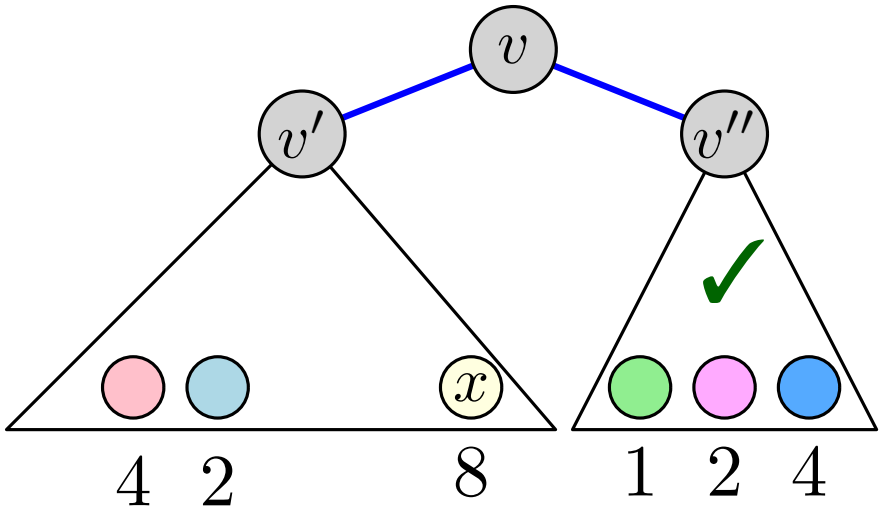
Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}w(v)$ or are leaves.

If the interval $[\frac{1}{3}w(v), \frac{2}{3}w(v)]$ contains a single segment, let x be the corresponding leaf





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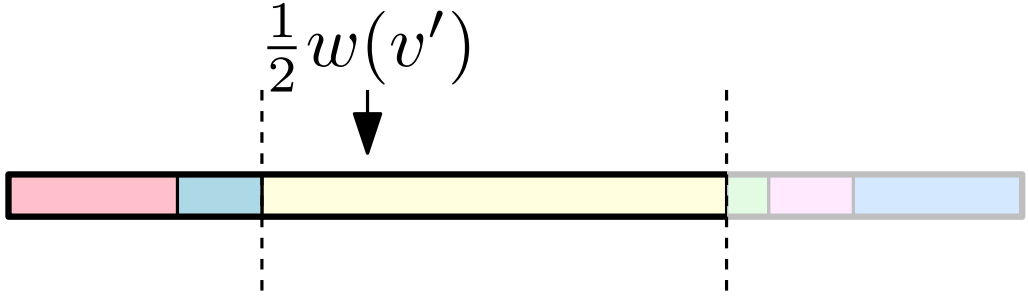

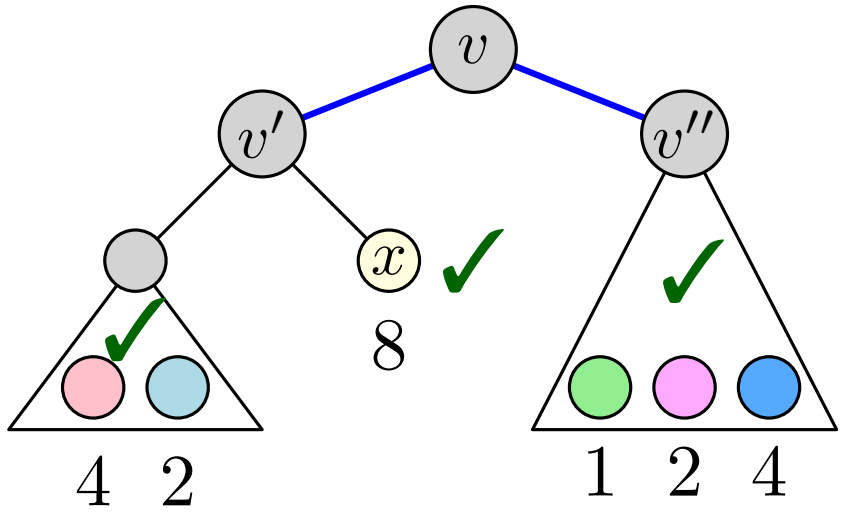

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- One child of v' is x and the other child weighs $\leq \frac{1}{2}w(v') \leq \frac{1}{2}w(v)$

Representing Tries

Weight-Balanced BSTs

Claim: All the grand-children u of v satisfy $w(u) \leq \frac{2}{3}(v)$ or are leaves.

Traversing two edges of a weight-balanced BST either:

- Brings us to the next node in the trie, i.e., we advance one character into P; or

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Overall space: $O(n)$ **Overall time:** $O(|P| + \log k)$

Representing Tries: Recap

	Space	**Query Time**
Array (dense)	$O(\|\Sigma\| \cdot n)$	$O(\|P\|)$
Array (sparse) / BST	$O(n)$	$O(\|P\| \log \|\Sigma\|)$
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Can we get rid of this term?

Optimal

Representing Tries: Recap

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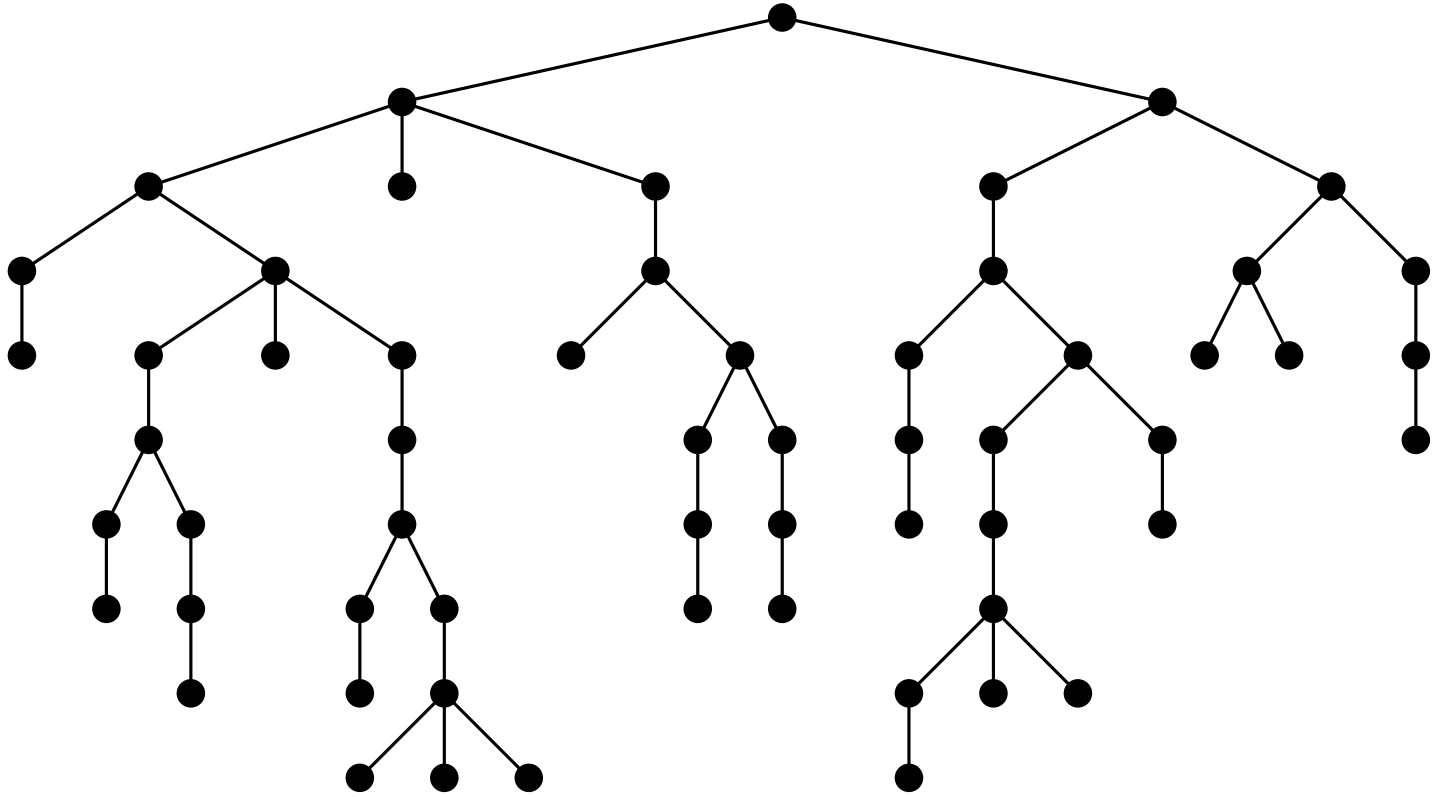
Optimal

Can we get rid of this term?

Almost...

Indirection

We can use a similar technique to the one we encountered while designing level ancestor oracles



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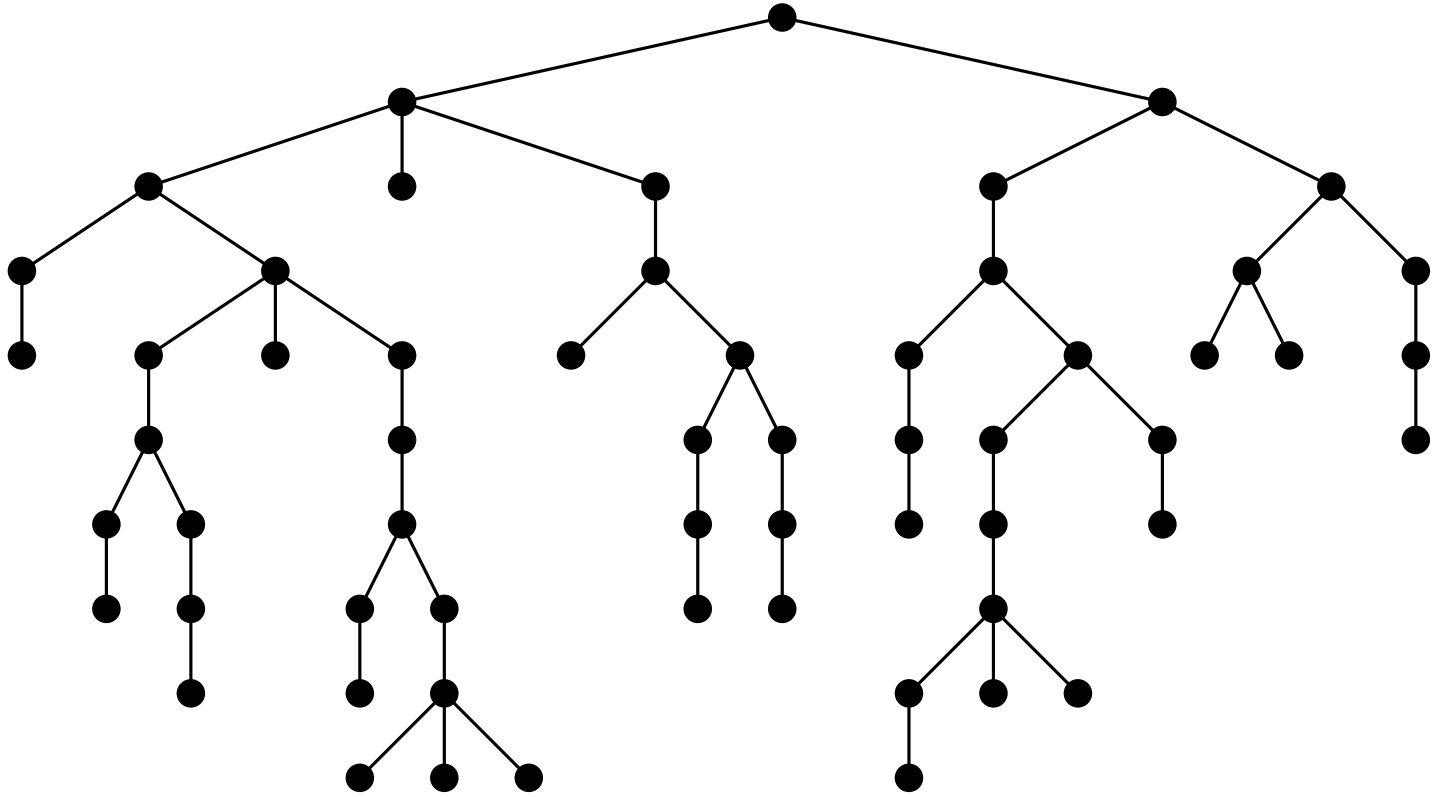

Find the set M of all maximally deep vertices with at least $|\Sigma|$ descendants

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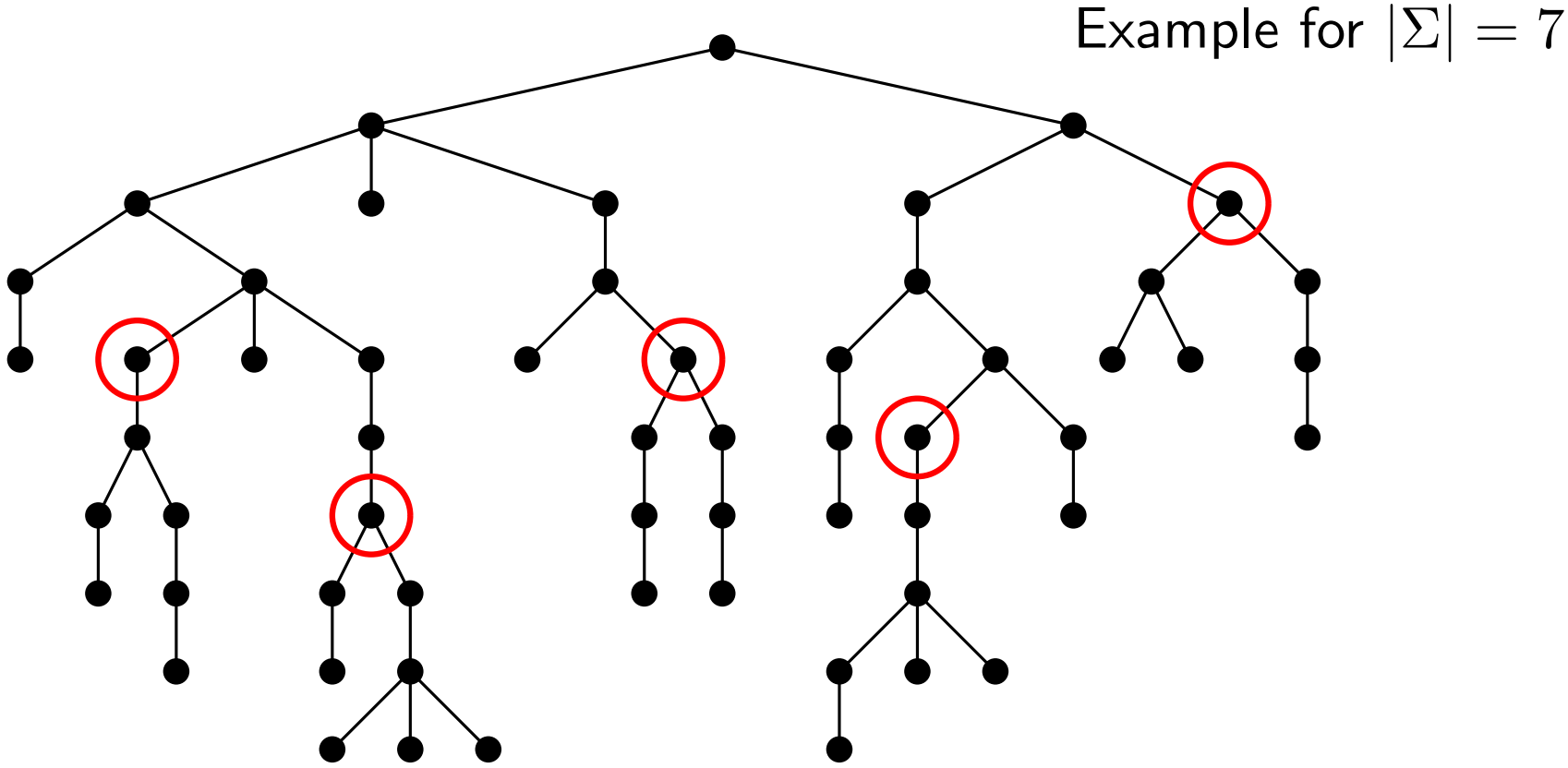

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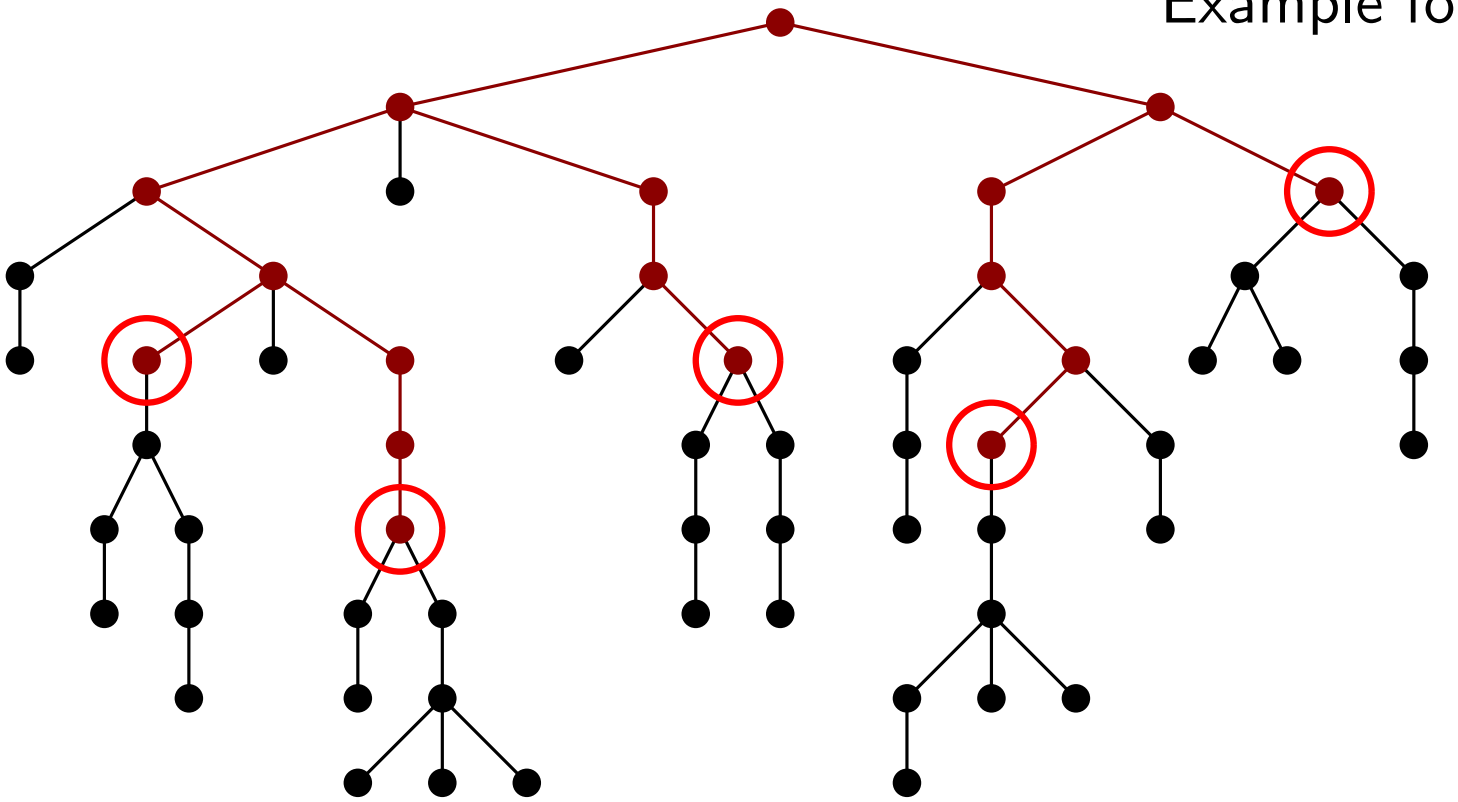

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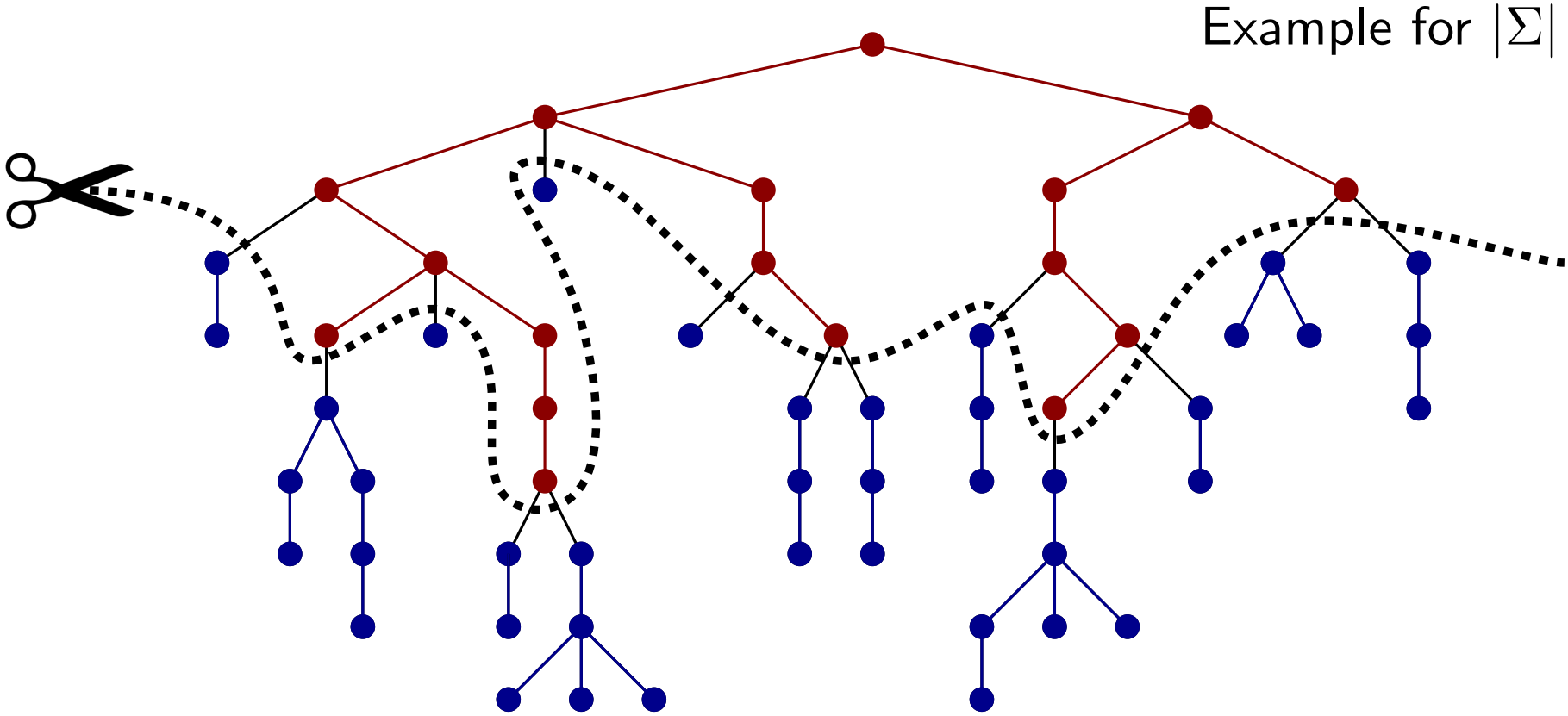

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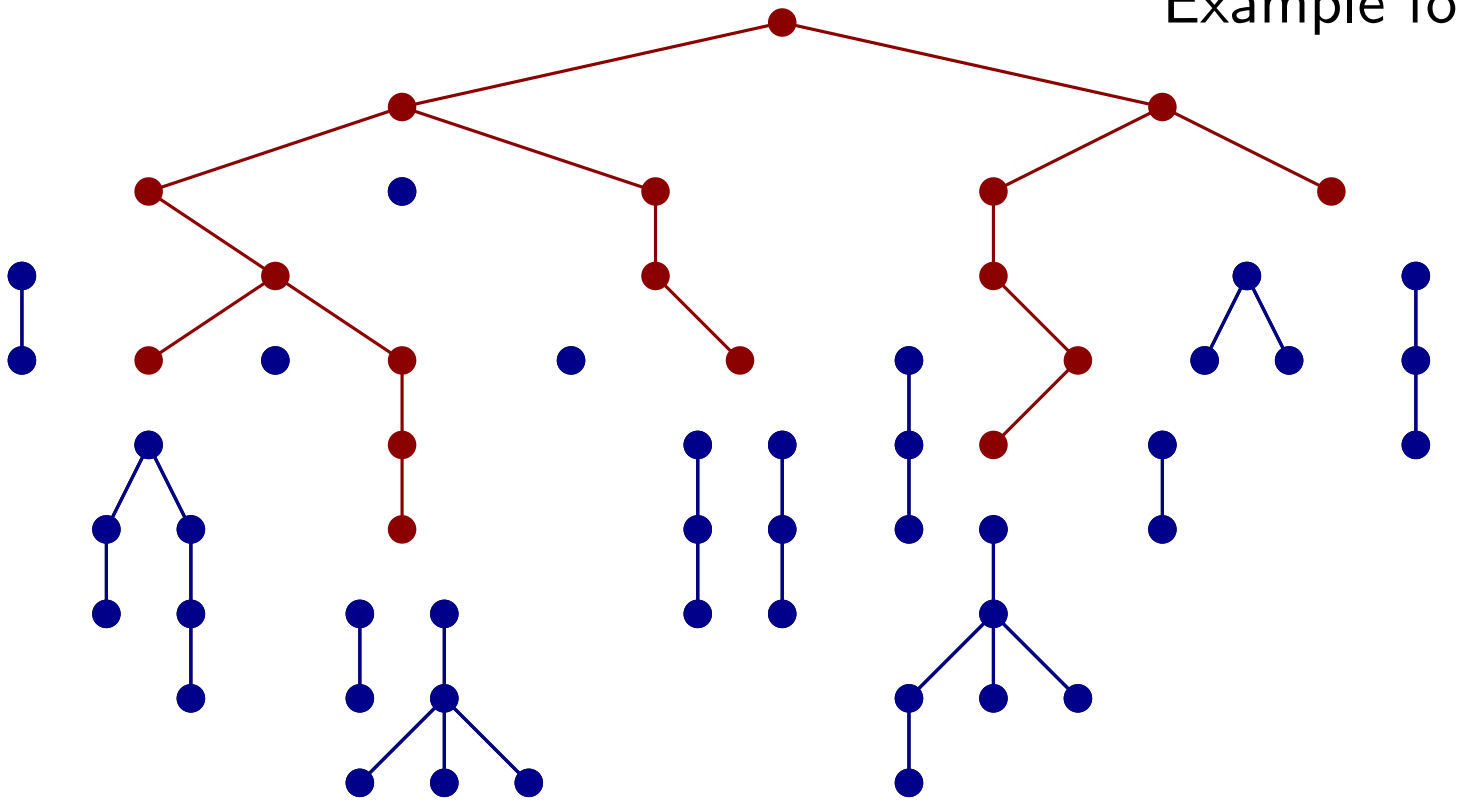

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Indirection

Storing the top tree:

The number of leaves of T' is at most $\frac{n}{|\Sigma|}$

Fact: A tree with ℓ leaves has at most $\ell - 1$ branching nodes (i.e., nodes with at least 2 children)

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- Store leaves using dense arrays

Space

$O(|\Sigma| \cdot \frac{n}{|\Sigma|}) = O(n)$

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Total space of all bottom trees: $O(n)$

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Total space of all bottom trees: $O(n)$

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Time to navigate a bottom tree: $O(|P| + \log |\Sigma|)$

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Can be made dynamic with a time complexity of $O(|T| + \log |\Sigma|)$ per insertion/deletion of T

Application: String Sorting

Sort a collection of k strings $T_1, T_2, \ldots, T_k$ over Σ

$$L = \max_{i=1,\ldots,k} |T_i|$$

Obs: A string comparison requires time $O(L)$.
Naive sorting algorithms take time $O(Lk \log k)$ or $O(L(k + |\Sigma|))$

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- Create an empty trie
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- An in-order visit of the trie returns the strings in lexicographic order

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Time: $O\left(\sum_{i=1}^{k} (|T_i| + \log |\Sigma|)\right)$

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$\Big\}\ O\left(n + k \log |\Sigma|\right))$

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$\Big\}\ O\left(n + k \log |\Sigma|\right))$

- An in-order visit of the trie returns the strings in lexicographic order — $O(n)$

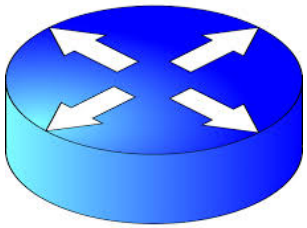
Overall time: $O\left(n + k \log |\Sigma|\right)$

Application: Packet Routing

Among all the destinations that match, a packet gets routed to the one with the most specific rule

Packet

Src: 192.168.42.10
Dst: 101.167.200.15



Routing Table

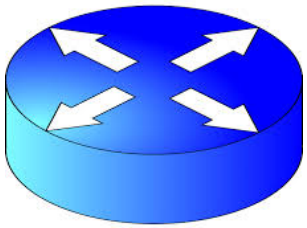
Destination	Interface
169.0.0.0/11	eth1
169.48.0.0/12	ppp0
169.128.0.0/10	eth1
169.160.0.0/11	eth0
96.0.0.0/3	tun1
96.0.0.0/6	tun0
100.0.0.0/8	eth0
127.0.0.0/8	lo
default	wlan0

Application: Packet Routing

Among all the destinations that match, a packet gets routed to the one with the most specific rule

Packet

Src: 192.168.42.10
Dst: 0110010110100111...



Routing Table

Destination	Interface
10101001000$	eth1
101010010011$	ppp0
1010100110$	eth1
10101001101$	eth0
011$	tun1
011000$	tun0
01100100$	eth0
01111111$	lo
$	wlan0

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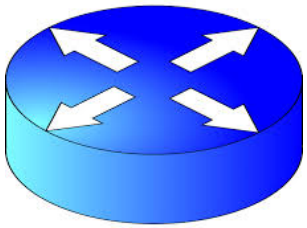
Packet

Src: 192.168.42.10

Dst: 011001011010011 1...



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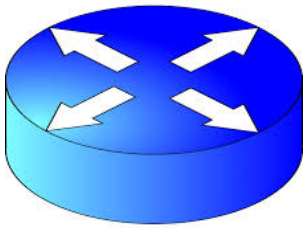
P



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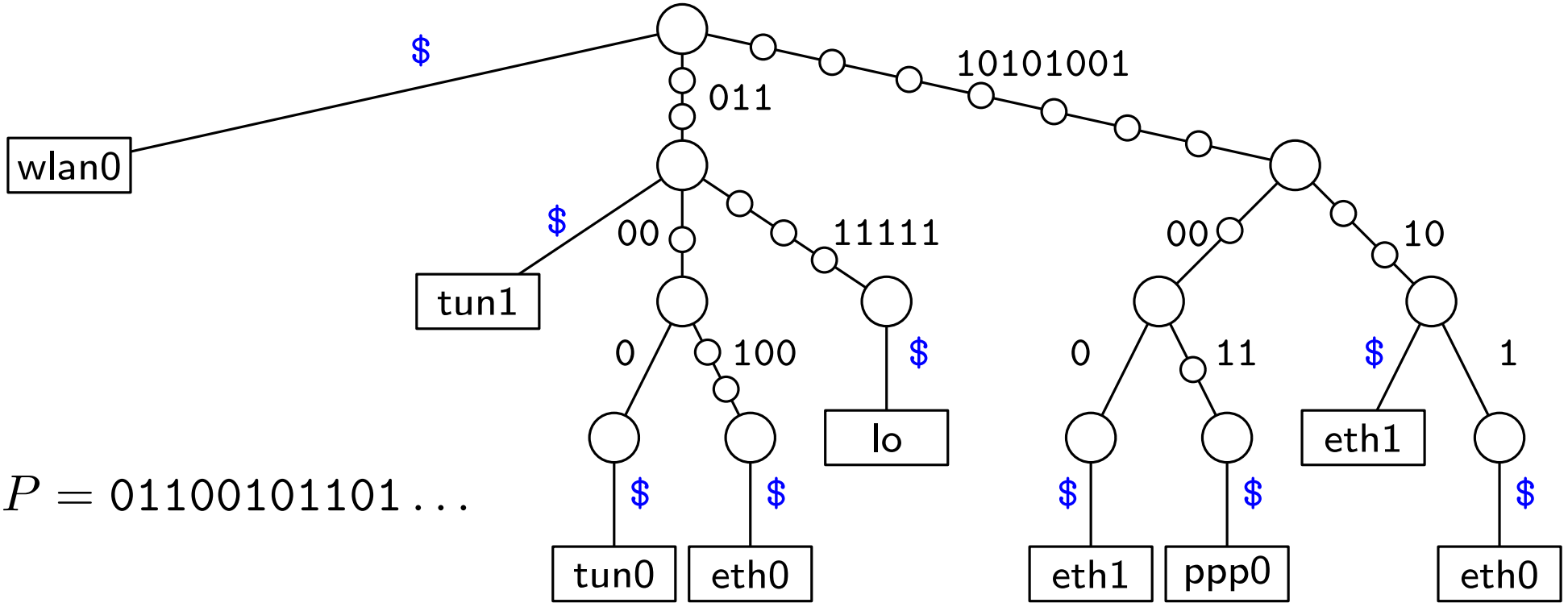
1010100110$	eth1
10101001101$	eth0
011$	tun1
011000$	tun0
01100100$	eth0
01111111$	lo
$	wlan0

Given a pattern P we want the longest string in our collection that appears as a prefix of P

Application: Packet Routing

Build a trie T with all the addresses in the routing table.

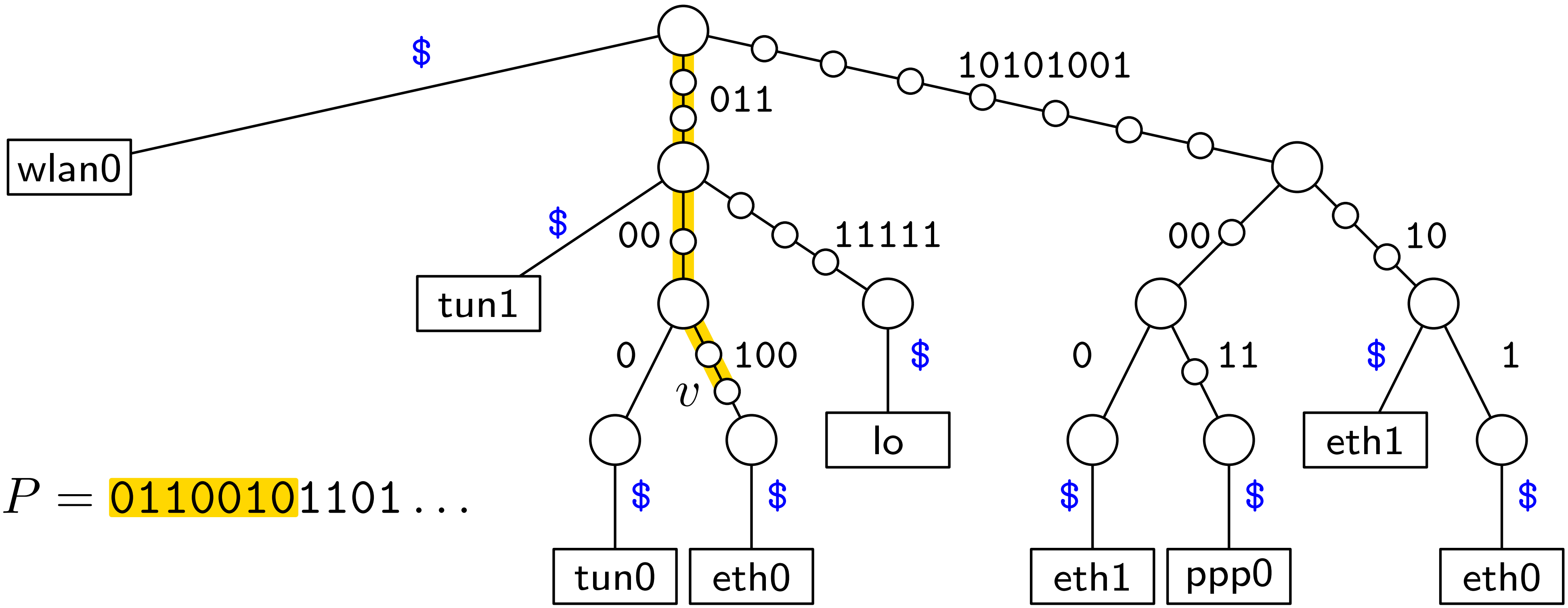


$P = 01100101101\ldots$

- Find the node v corresponding to the maximal prefix that matches P

Application: Packet Routing

Build a trie T with all the addresses in the routing table.



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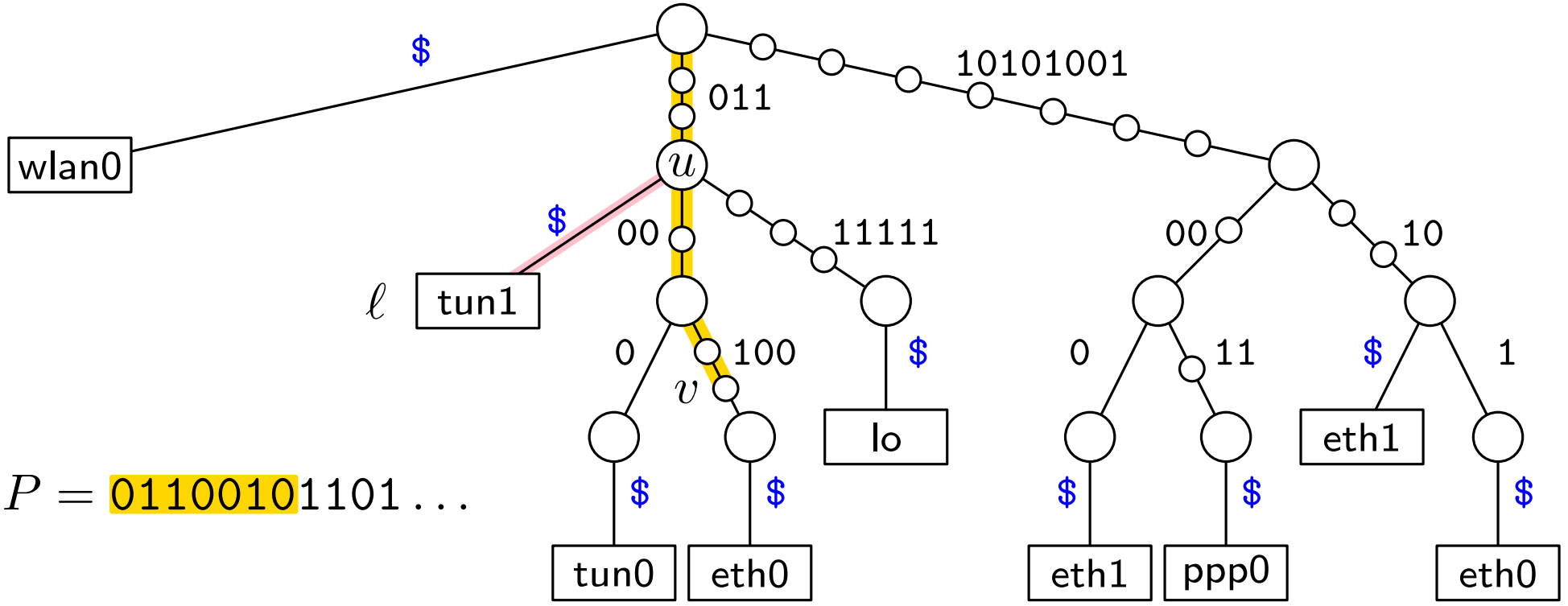

$P = 01100101101\ldots$

- Find the node v corresponding to the maximal prefix that matches P
- Walk up the tree searching for the deepest ancestor u of v incident to a "$" edge towards a leaf ℓ

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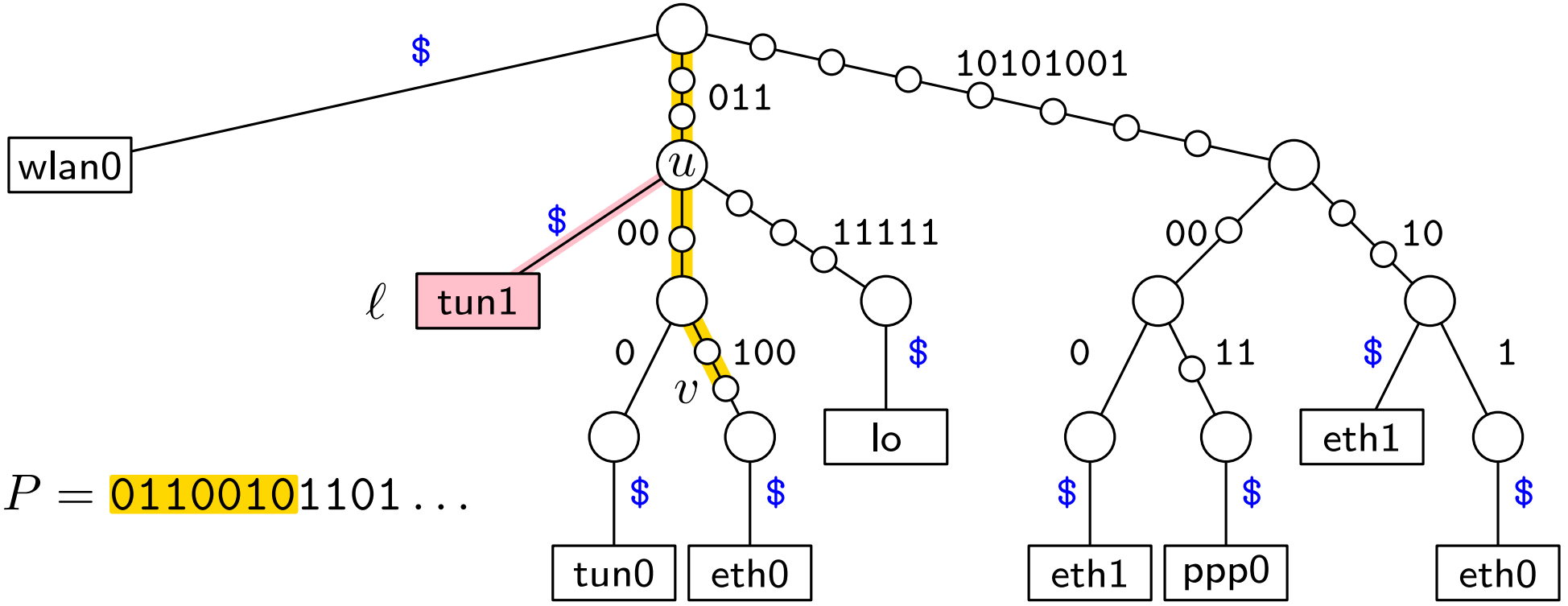

- Find the node v corresponding to the maximal prefix that matches P
- Walk up the tree searching for the deepest ancestor u of v incident to a "$" edge towards a leaf ℓ
- Route the packet towards the interface stored in ℓ

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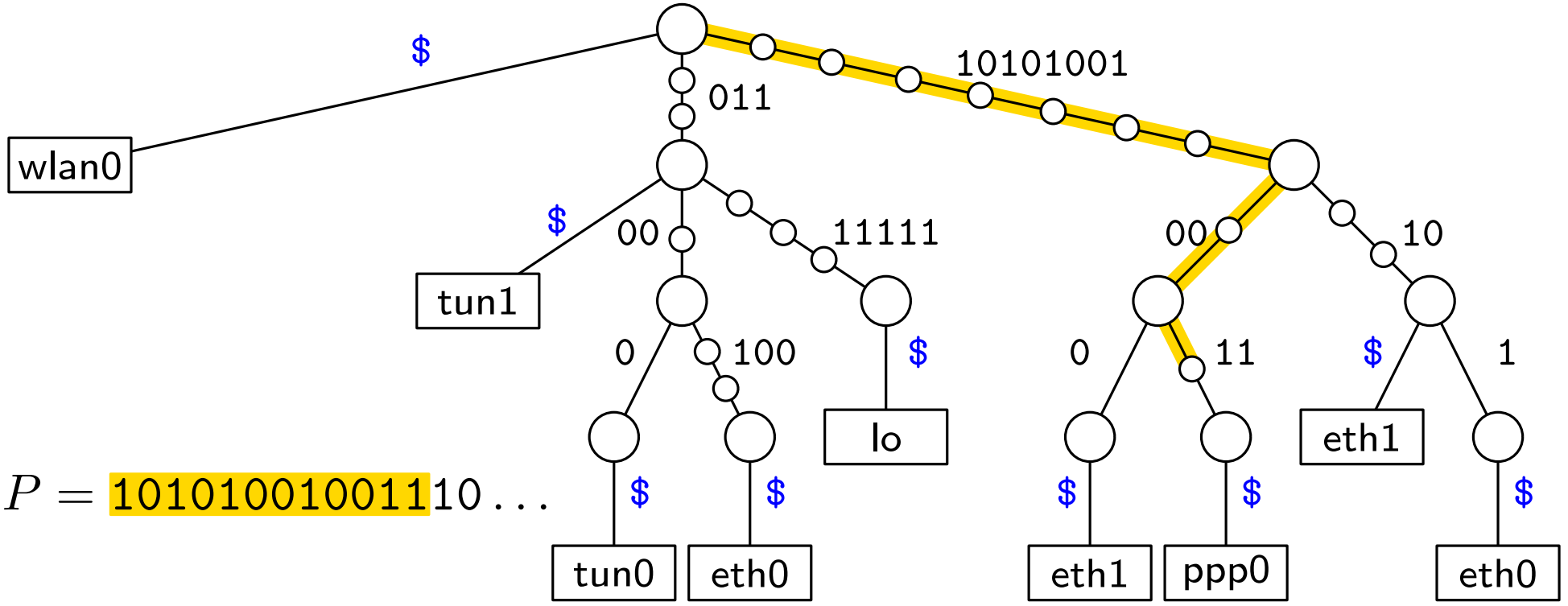

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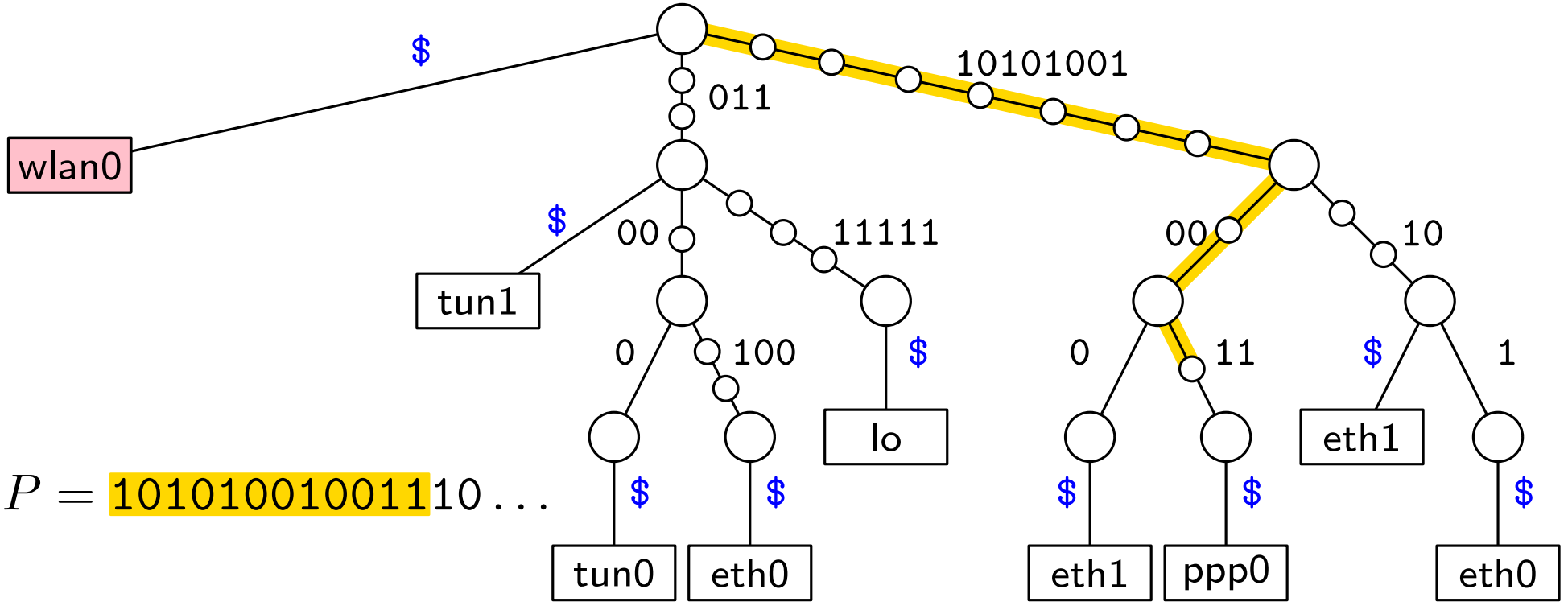

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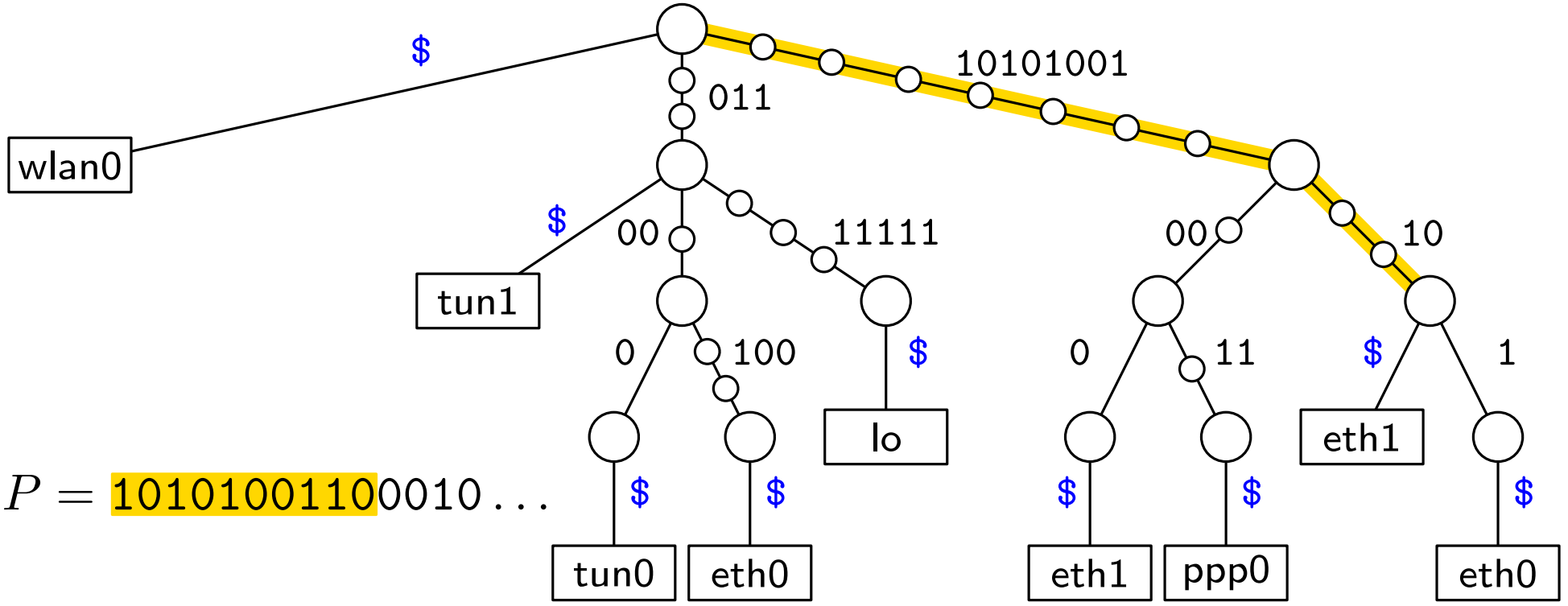
Application: Packet Routing

Build a trie T with all the addresses in the routing table.



$P = 10101001100010\ldots$

- Find the node v corresponding to the maximal prefix that matches P
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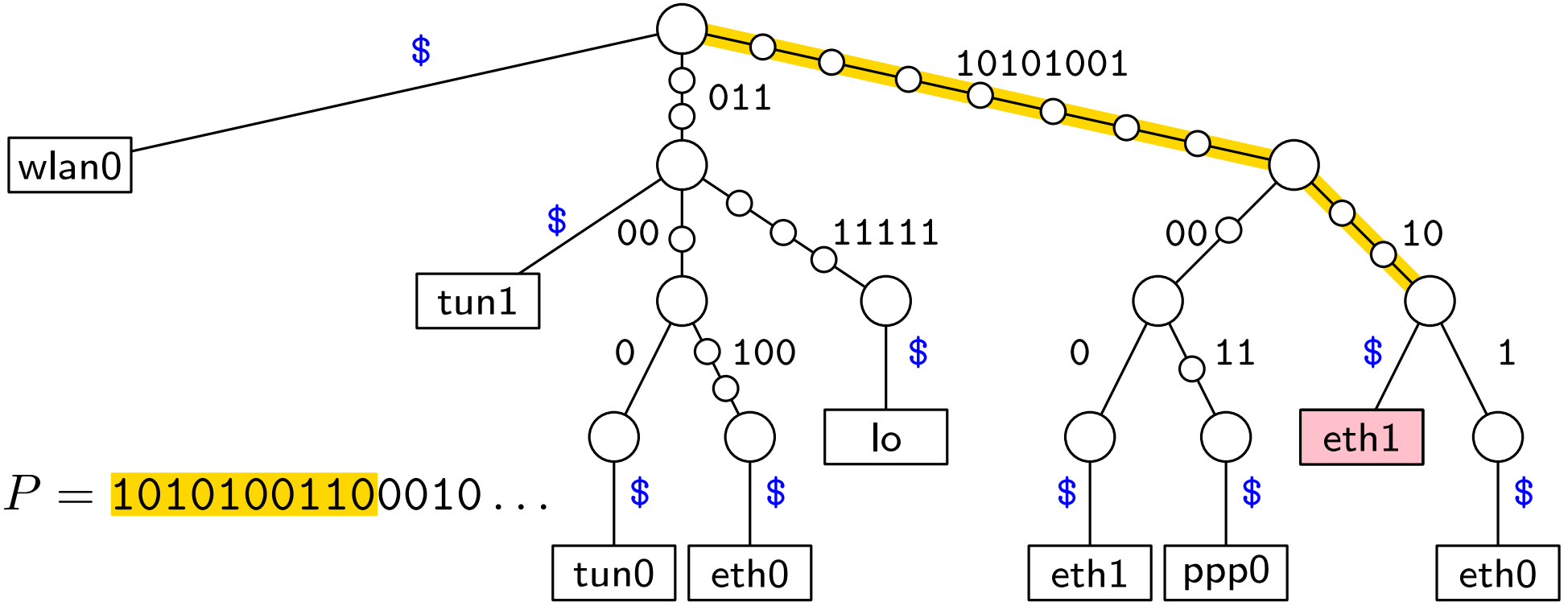

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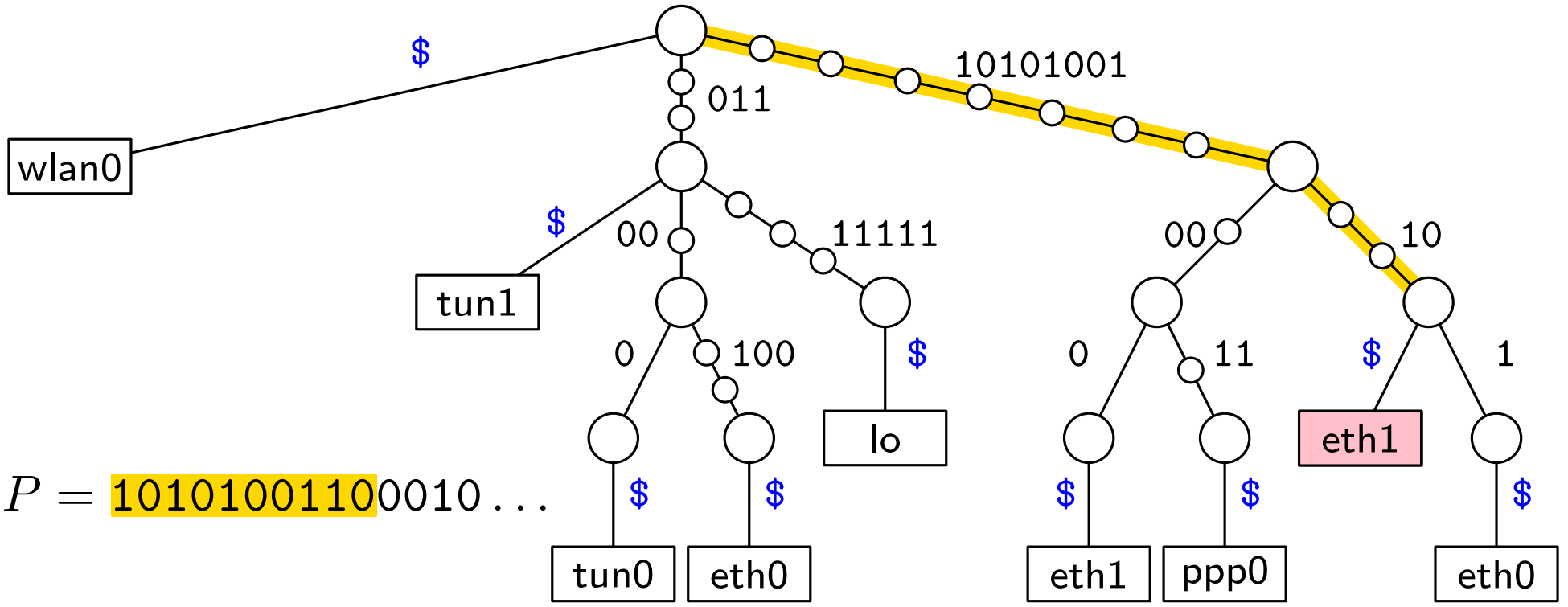

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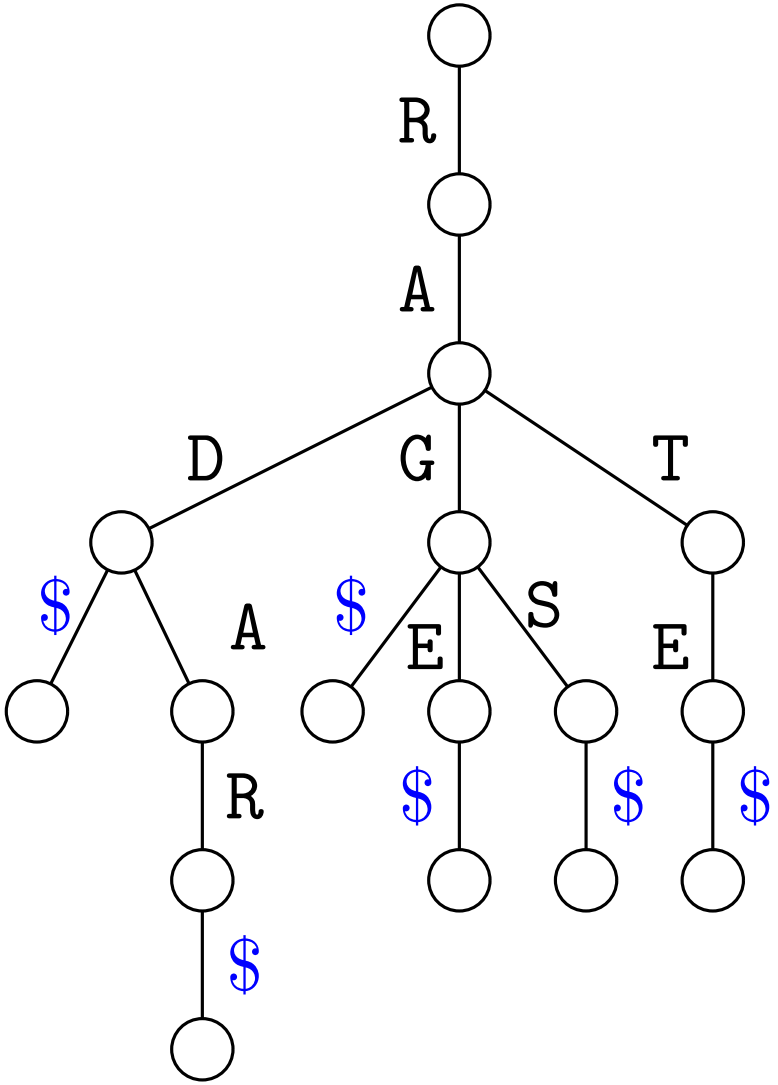
Time: O(address length)

Compressed Tries (Radix Trees)

Contract non-branching paths to a single edge labelled with the corresponding substring



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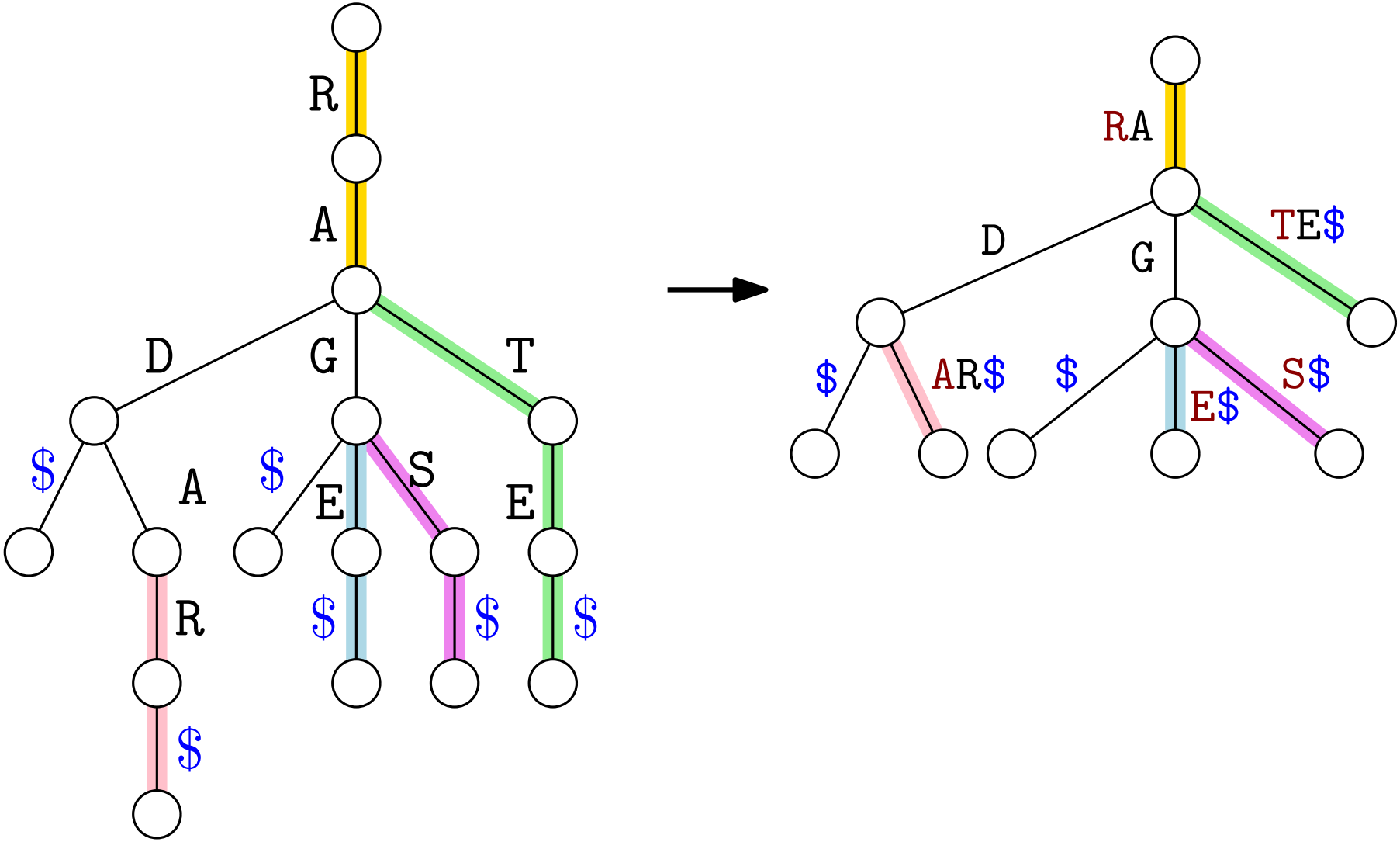

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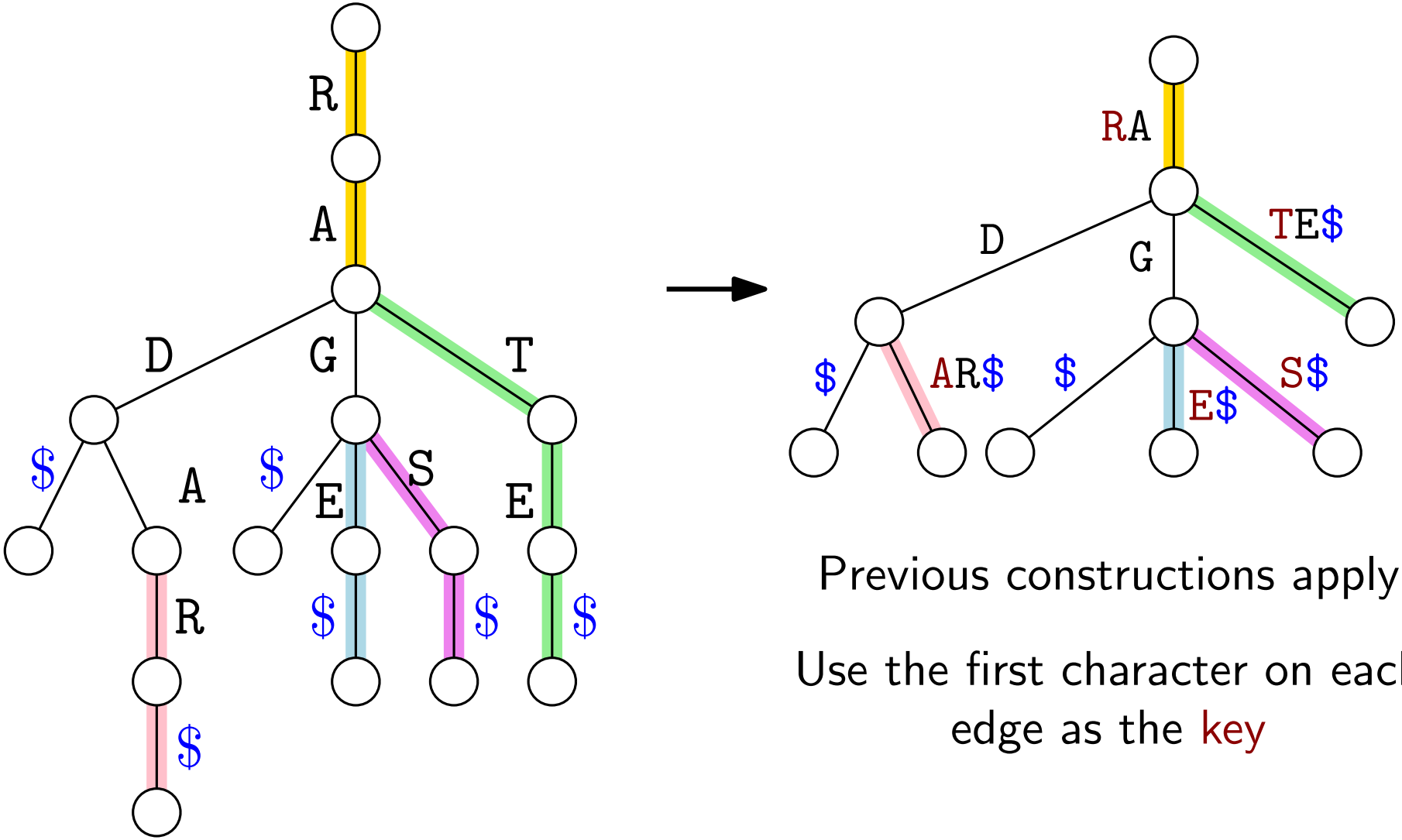

Previous constructions apply

Use the first character on each edge as the key

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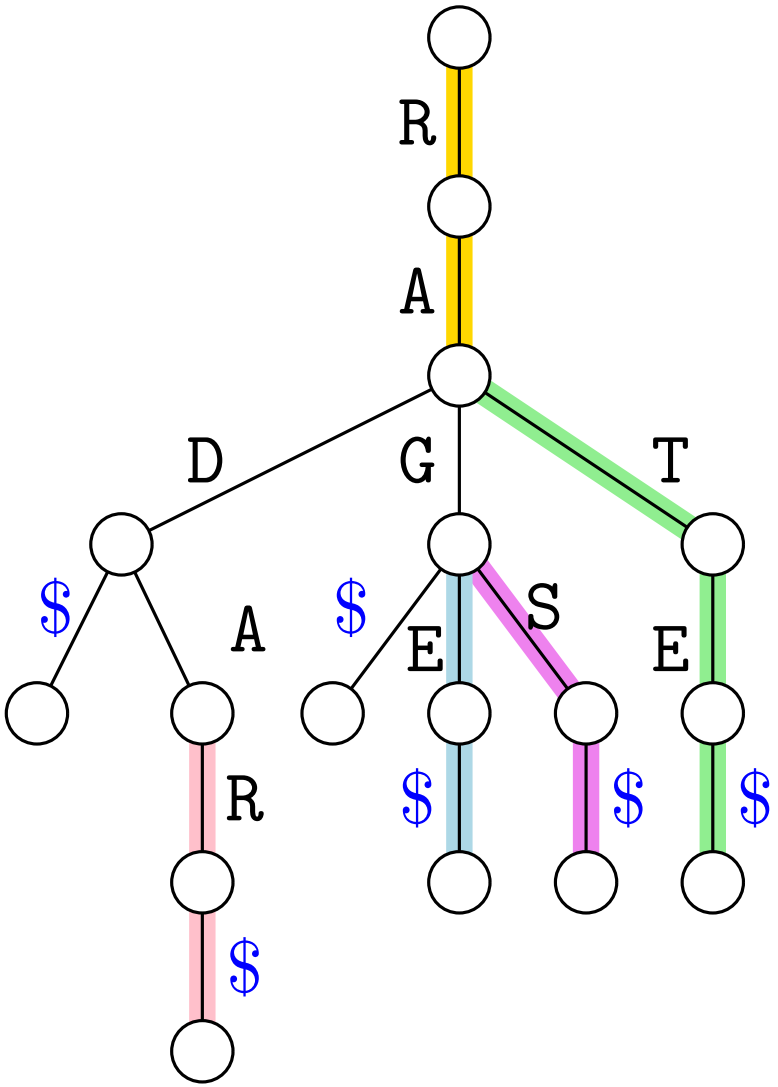
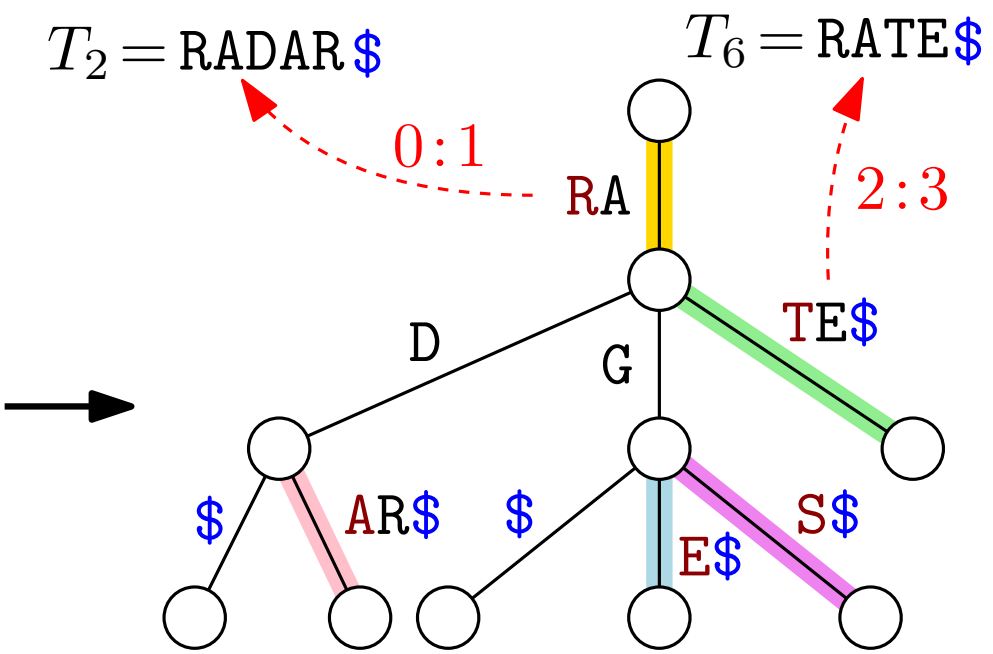




Previous constructions apply

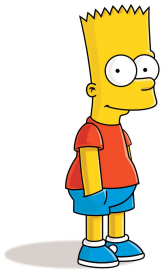
Use the first character on each edge as the key

Store edge labels as indices in the input strings

Suffix Trees

Back to String Matching

Problem: Given an alphabet Σ, a *text* $T \in \Sigma^*$ and a *pattern* $P \in \Sigma^*$, find some occurrence/all occurrences of P in T.



$$\Sigma = \{\texttt{A}, \texttt{B}, \ldots, \texttt{Z}, \texttt{a}, \texttt{b}, \ldots, \texttt{z}, \texttt{␣}\}$$

$$T = \texttt{Bart␣played␣darts␣at␣the␣party}$$

$$P = \texttt{art}$$

Want: A data structure that can preprocesses T and answer string matching queries

Suffix Trees

The **suffix tree** of T is the compressed trie of all the suffixes of $T\$$

$\Sigma = \{\mathtt{A}, \mathtt{B}, \mathtt{N}, \mathtt{S}\}$ $\quad$ $T = \mathtt{BANANAS}\$$

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01234567

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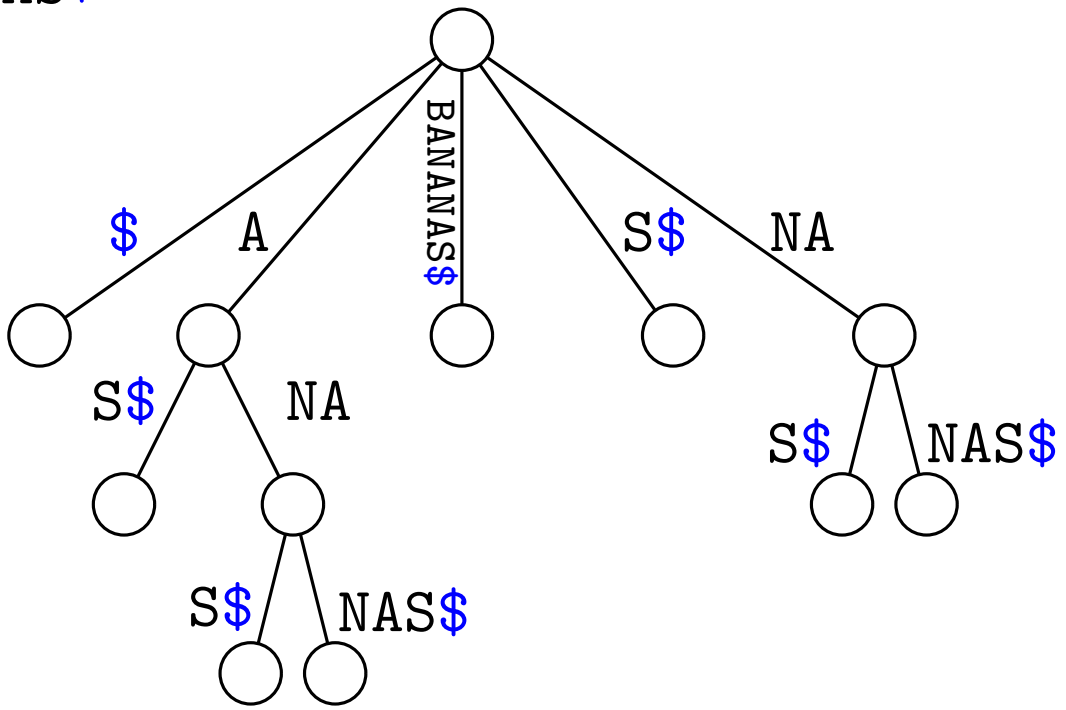

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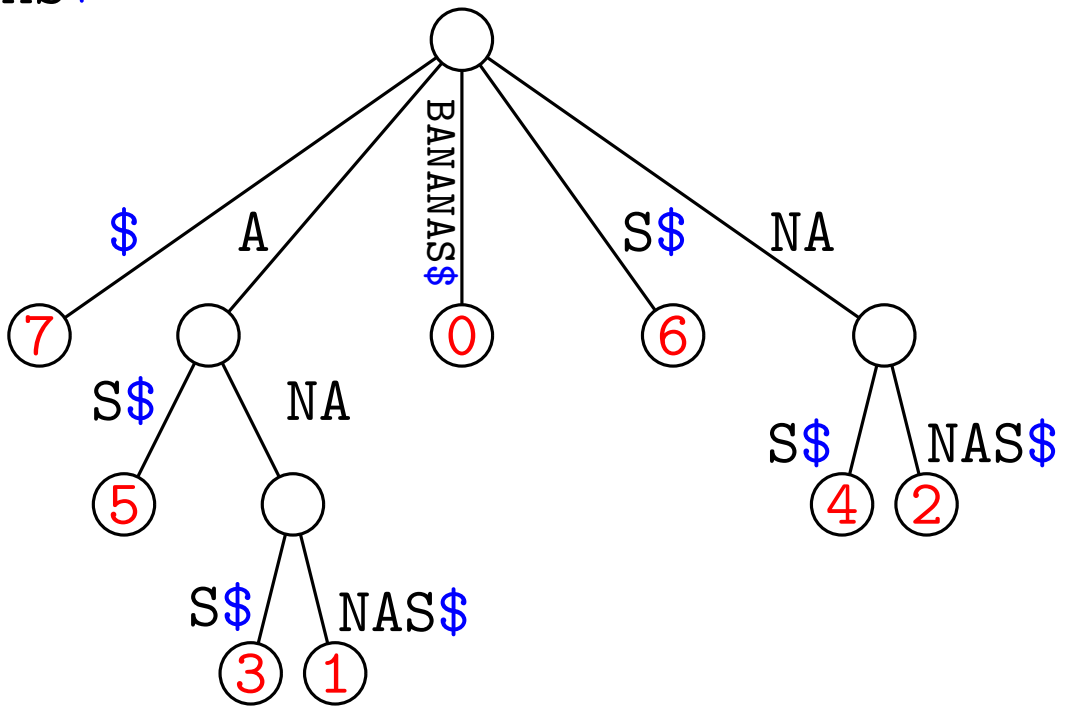

Label edges with indices into T

Label leaves with the index of the start of the corresponding suffix

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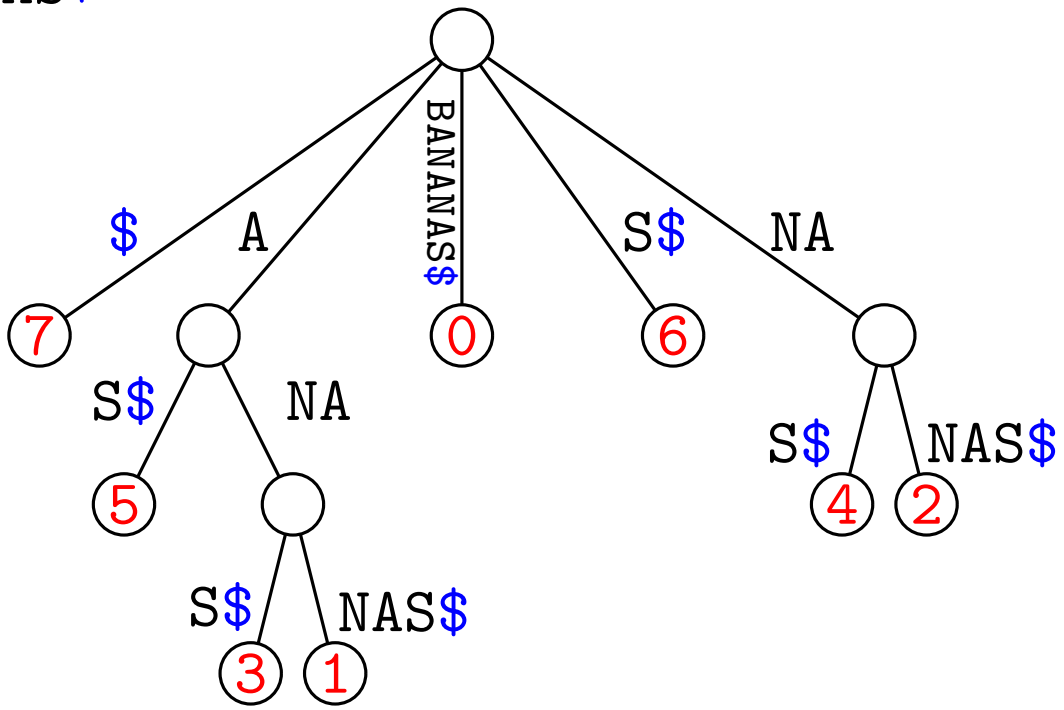


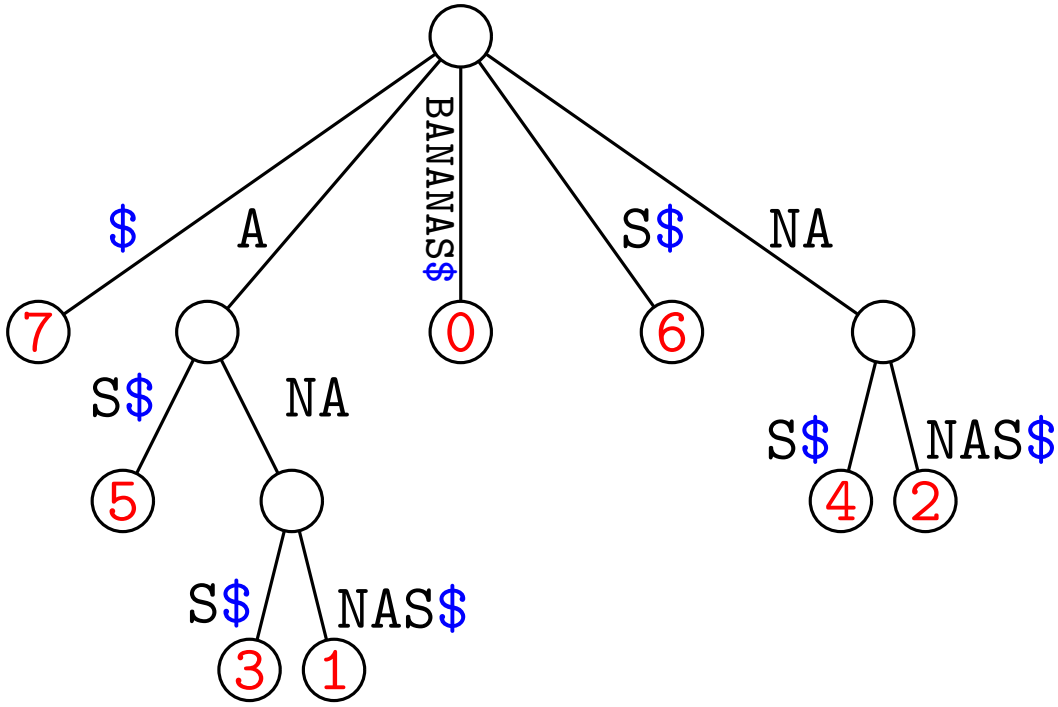
Label edges with indices into T

Label leaves with the index of the start of the corresponding suffix

Space: $\quad O(\#\text{ nodes}) = O(\#\text{ leaves}) = O(|T|)$

Applications: String Matching

01234567
$T = \texttt{BANANAS\$}$



$P = \texttt{NA}$

Searching for a pattern P returns a compact representation of **all** occurrences of P in T

- Find the node v corresponding to P
- The occurrences of P are all and only the leaves in the subtree of v

Applications: String Matching

01234567
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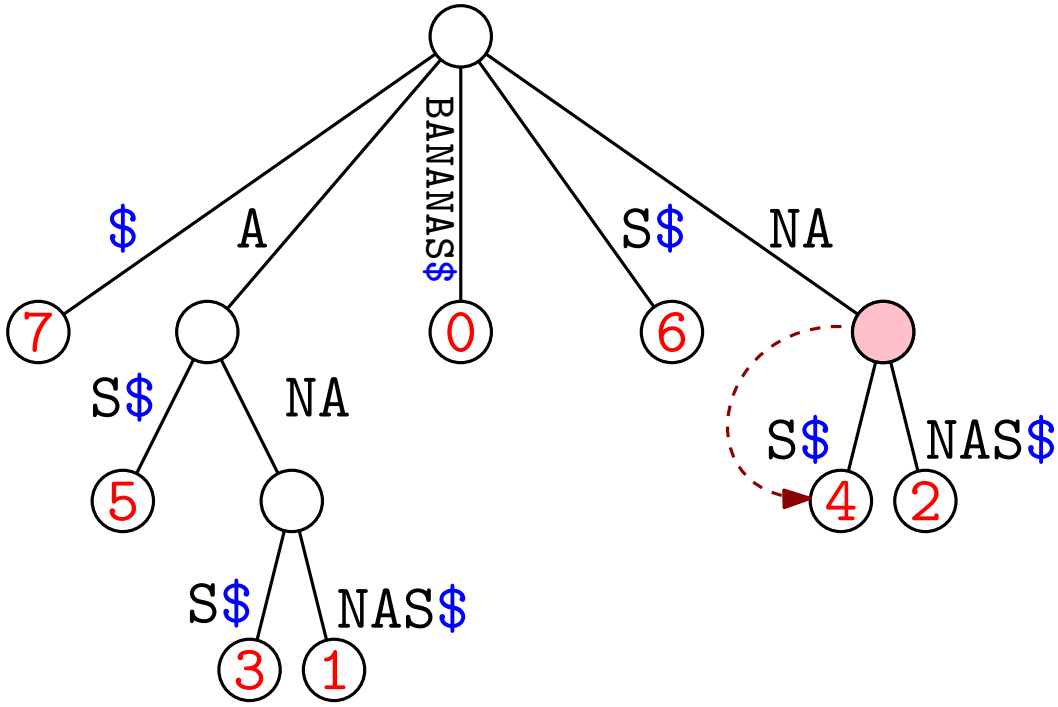

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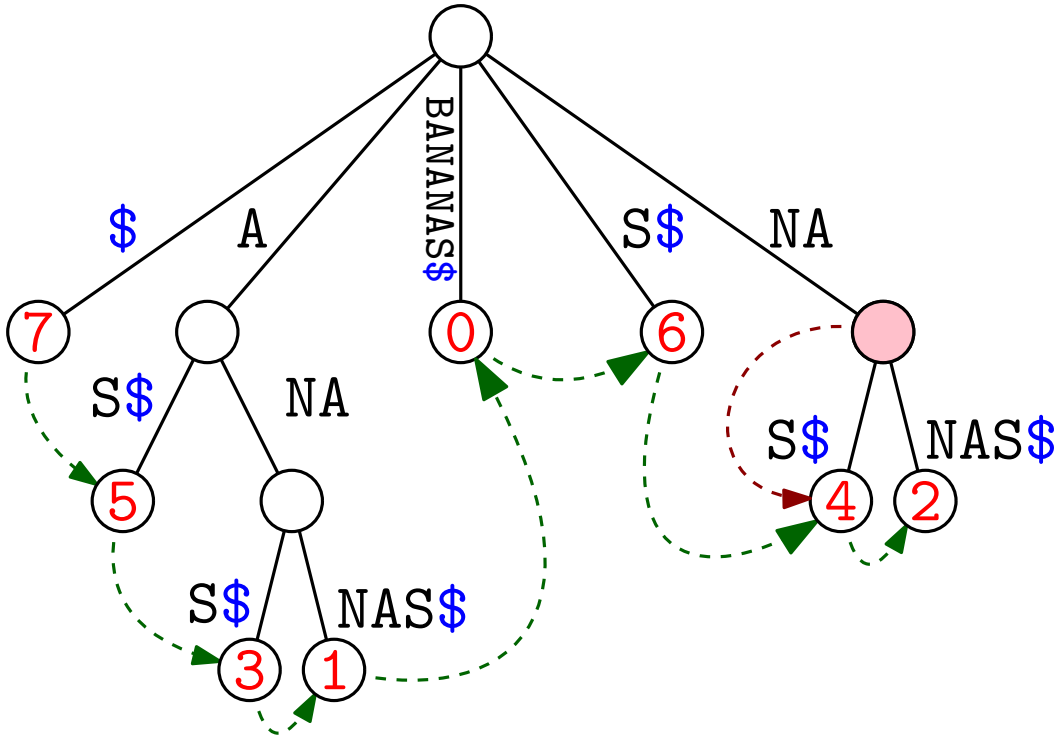

Searching for a pattern P returns a compact representation of **all** occurrences of P in T

- Find the node v corresponding to P
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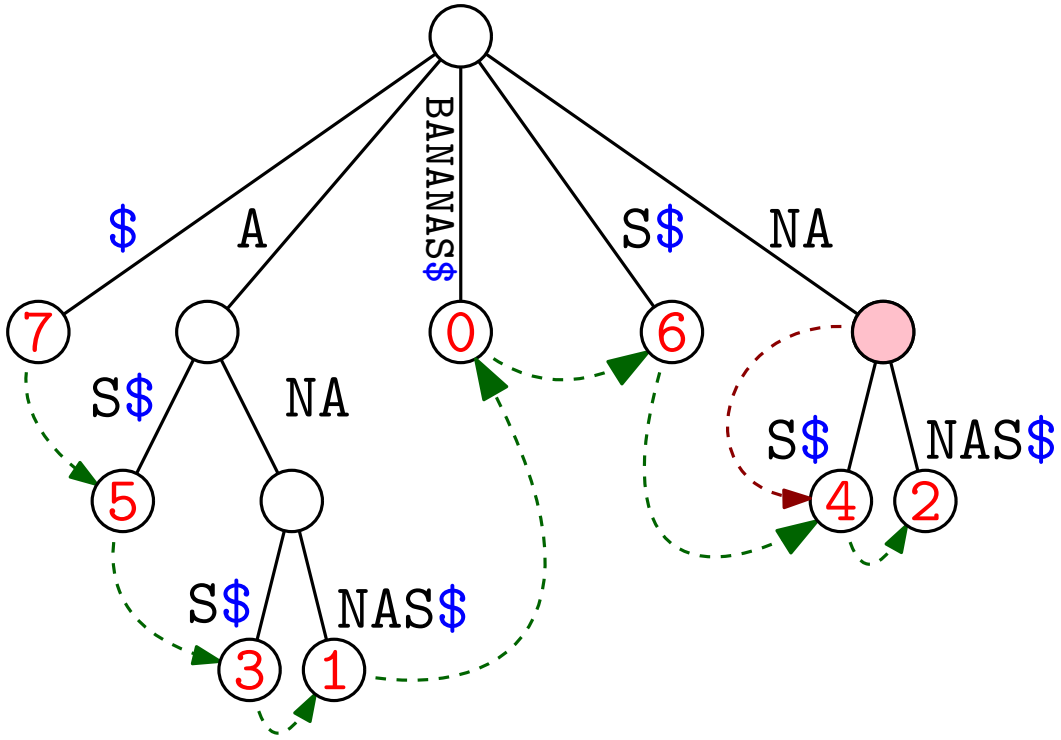
- Arrange leaves in a linked list to find the next match in $O(1)$ time

Applications: String Matching

$T =$ BANANAS$

01234567



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Applications: String Matching

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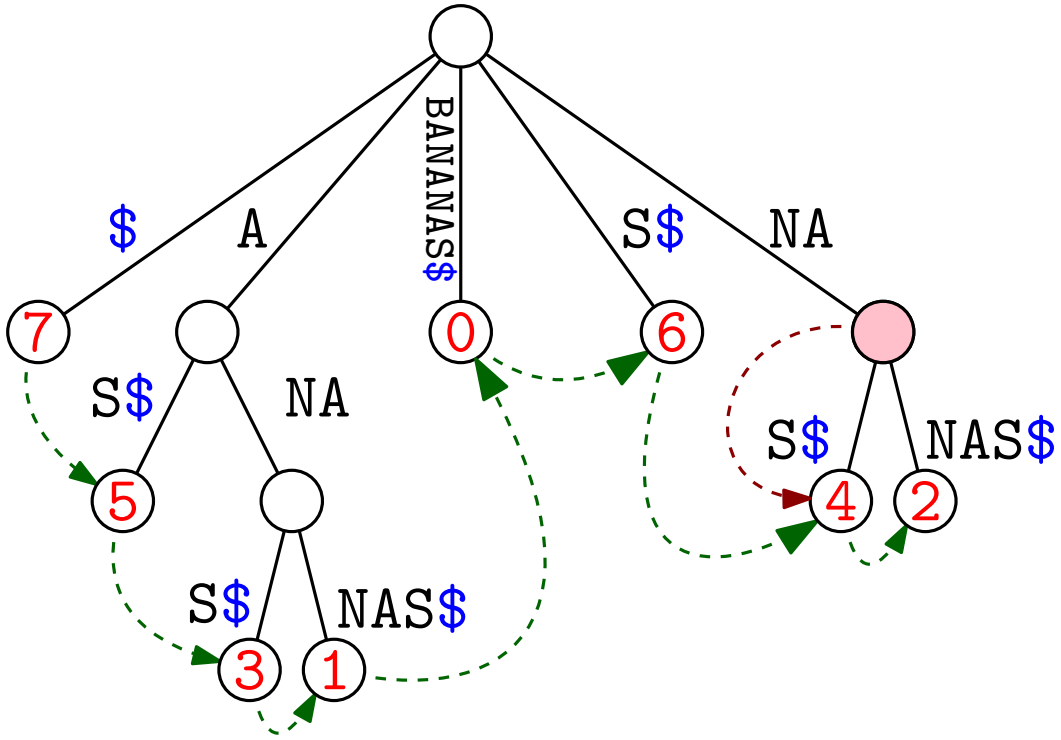

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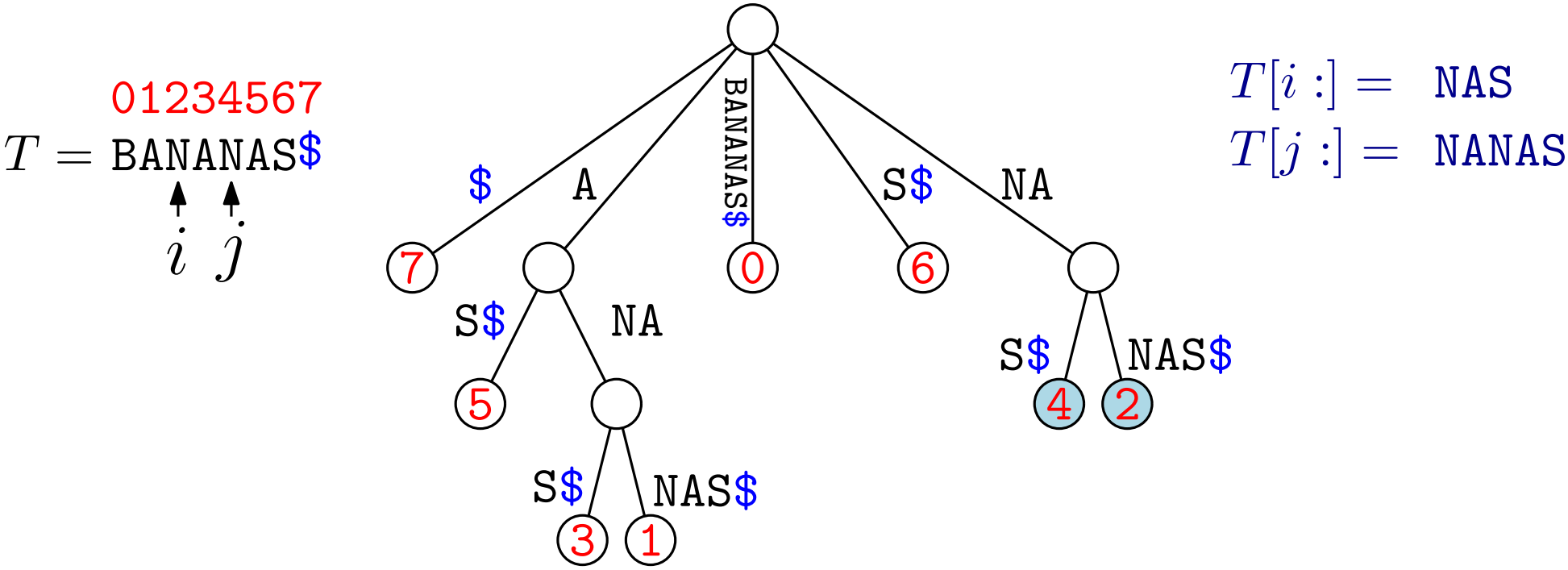
Time: $O(|P| + \log|\Sigma| + \#\text{desired matches})$

Number of matches in time $O(|P| + \log|\Sigma|)$

Each vertex stores the # of leaves in its subtrees

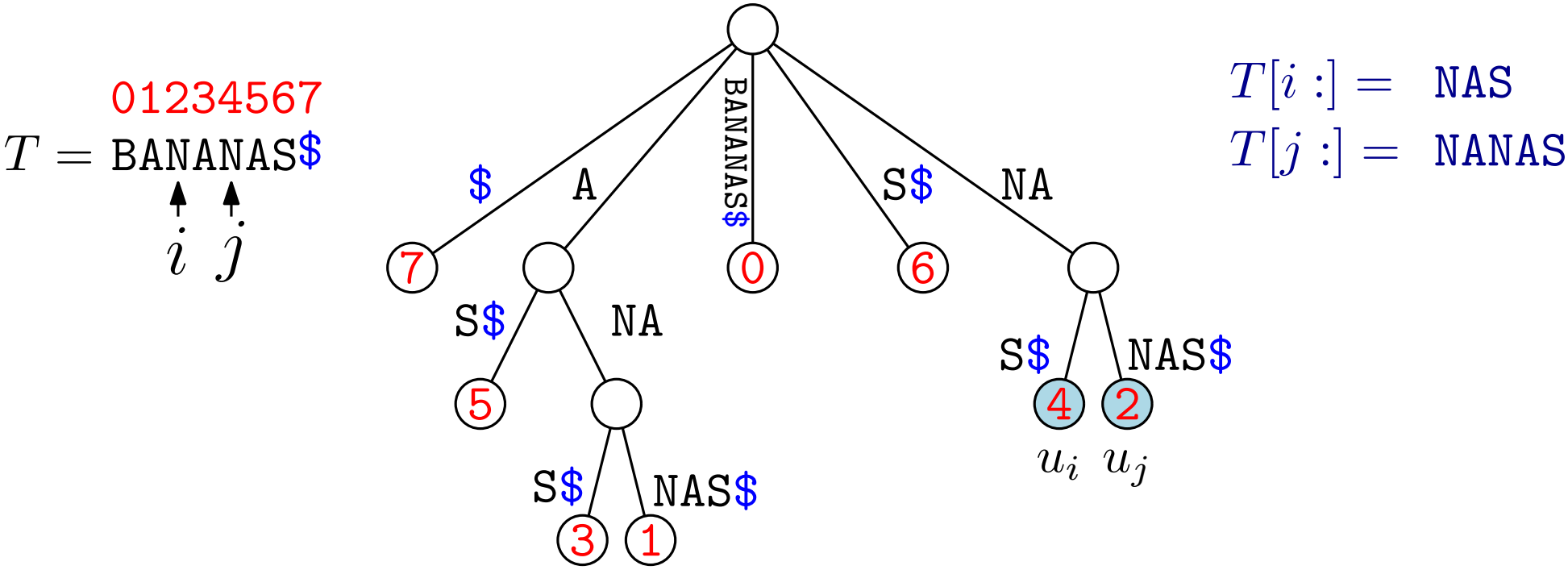
Applications: Longest Common Prefix



Given indicies i and j, find the longest common prefix of $T[i:]$ and $T[j:]$

- Look at the leaves u_i, u_j corresponding to $T[i:]$ and $T[j:]$

Applications: Longest Common Prefix



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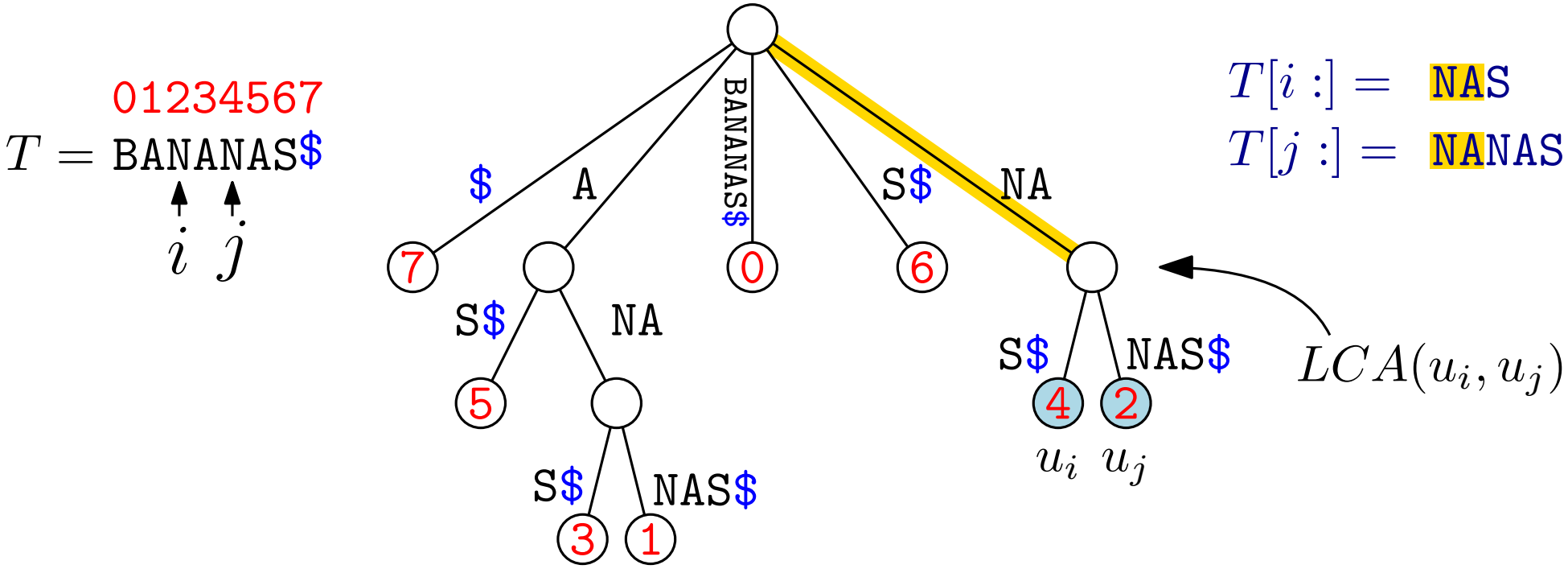

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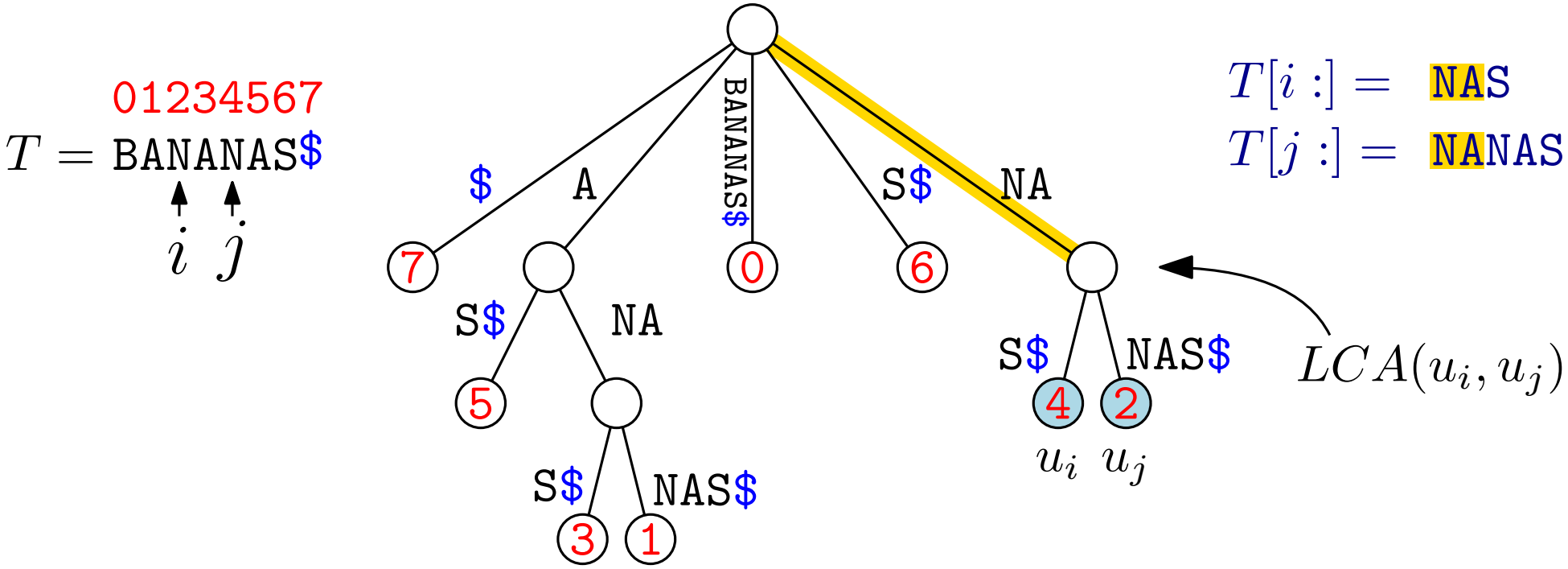

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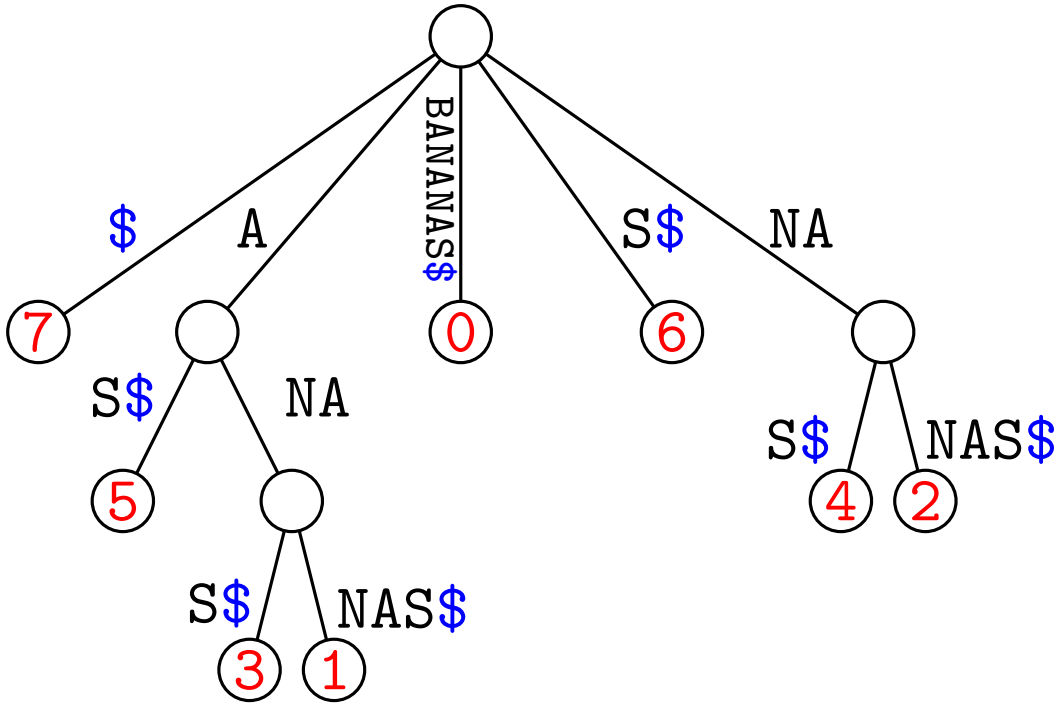
  We already know how to answer LCA queries in constant time!

Applications: Longest Repeated Substring

01234567
$T =$ BANANAS$



Find the longest string that appears at least twice in T as a substring:

Applications: Longest Repeated Substring

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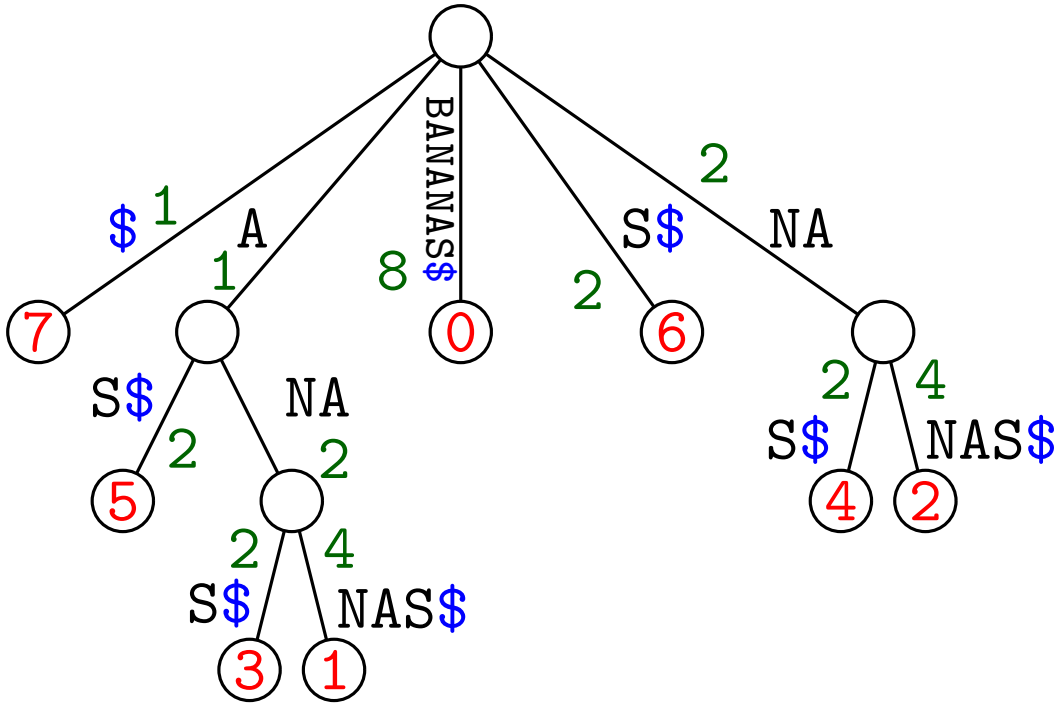
01234567



Find the longest string that appears at least twice in T as a substring:

- Assign a length to each edge equal to the number of symbols in its label

Applications: Longest Repeated Substring

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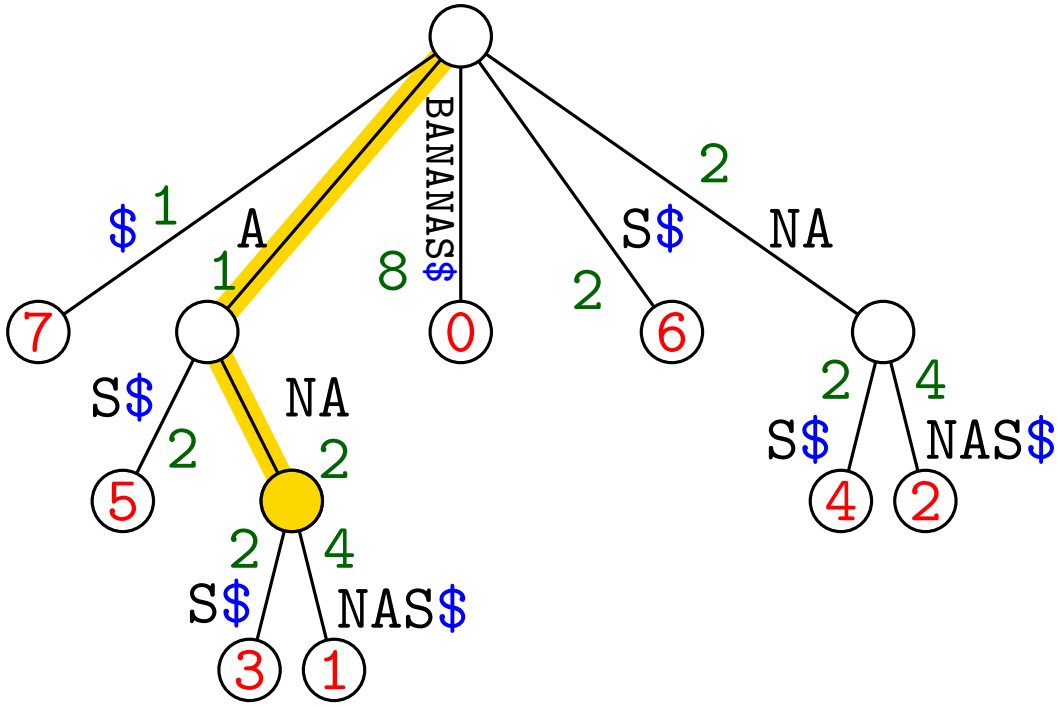

Find the longest string that appears at least twice in T as a substring:

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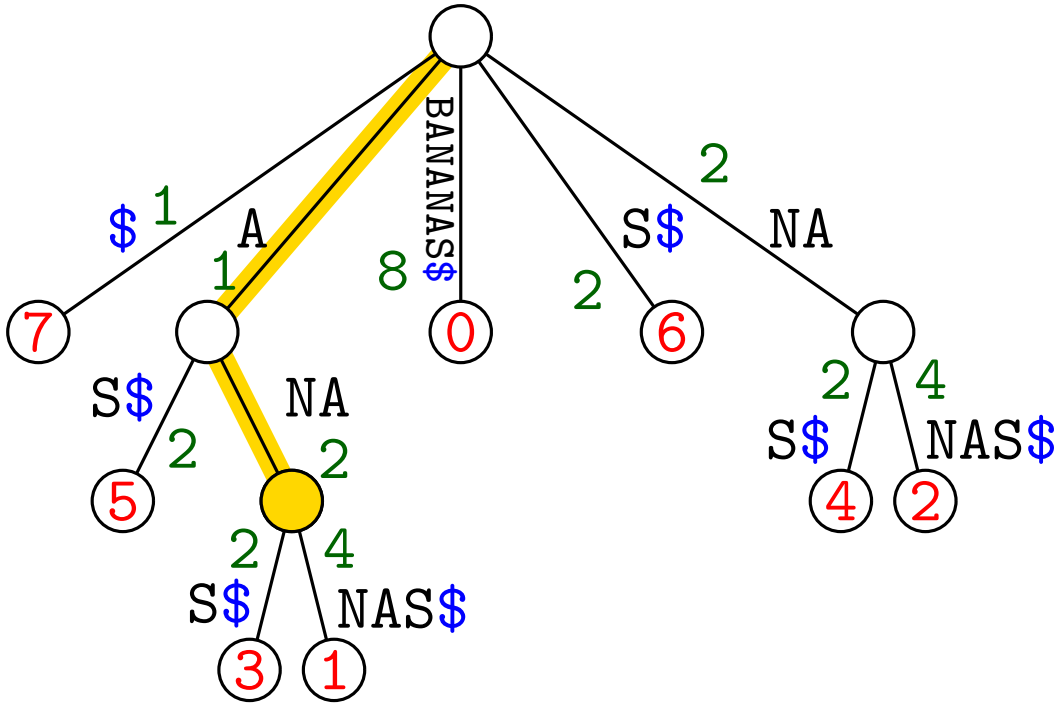

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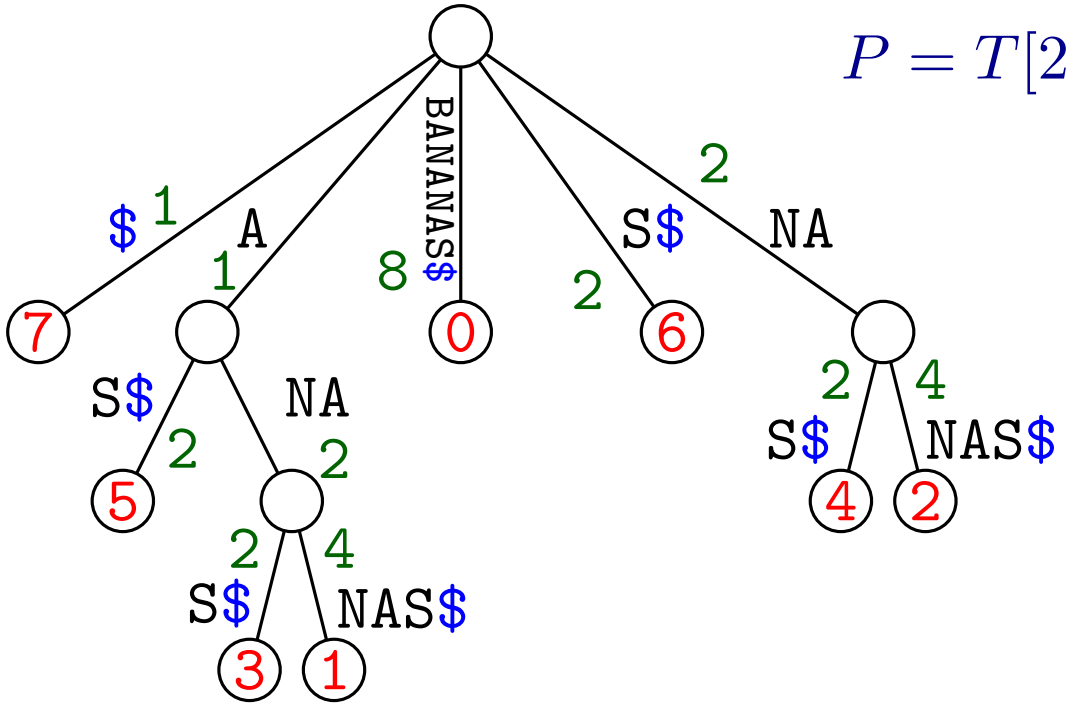
Time: $O(|T|)$

Applications: Finding Additional Matches

$T =$ BANANAS$

01234567

$P = T[2:3] =$ NA



Given an occurrence $T[i:j]$ of P in T, find all other occurrences of P:

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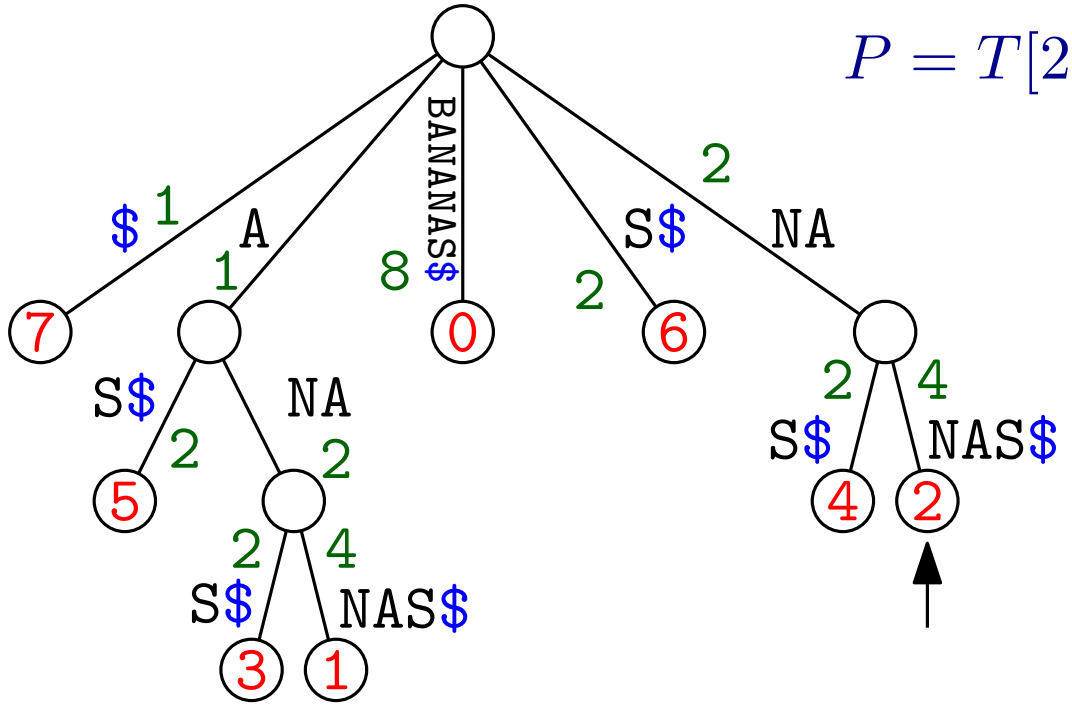

Given an occurrence $T[i:j]$ of P in T, find all other occurrences of P:

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Applications: Finding Additional Matches

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01234567

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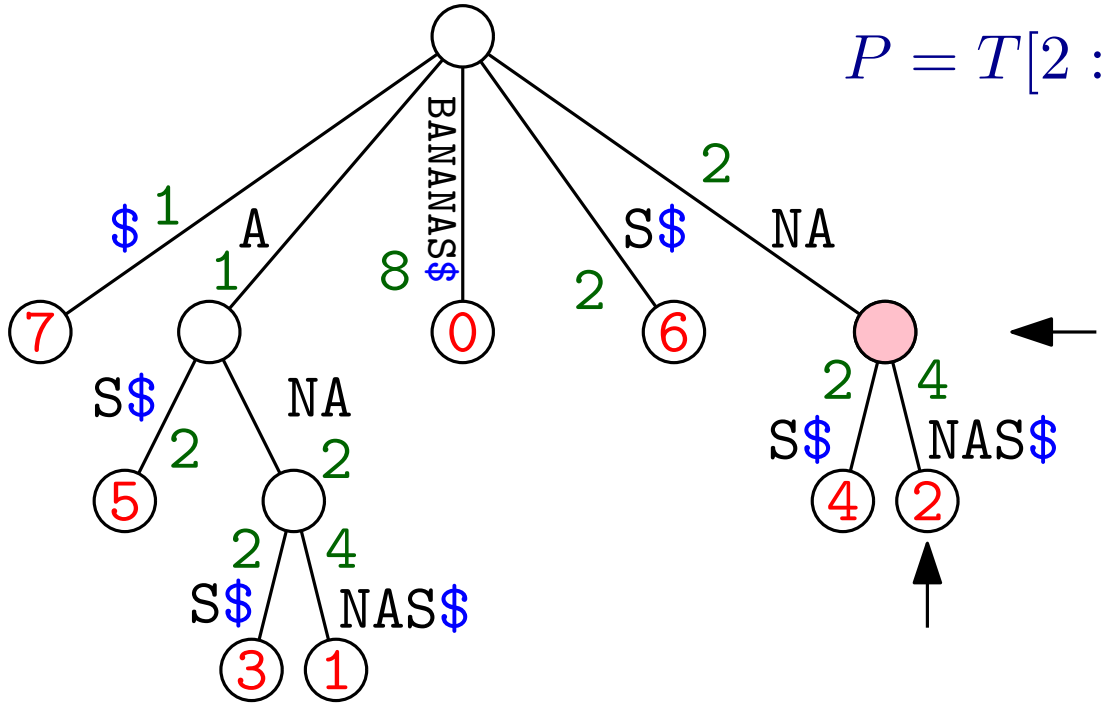

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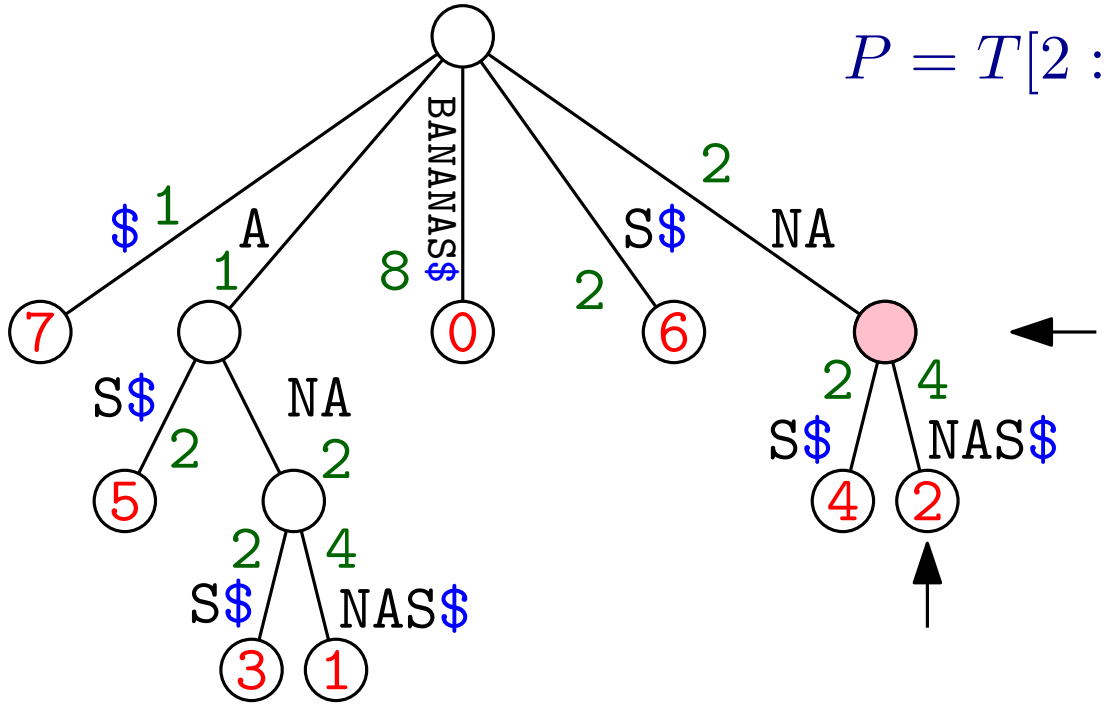

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Applications: Finding Additional Matches

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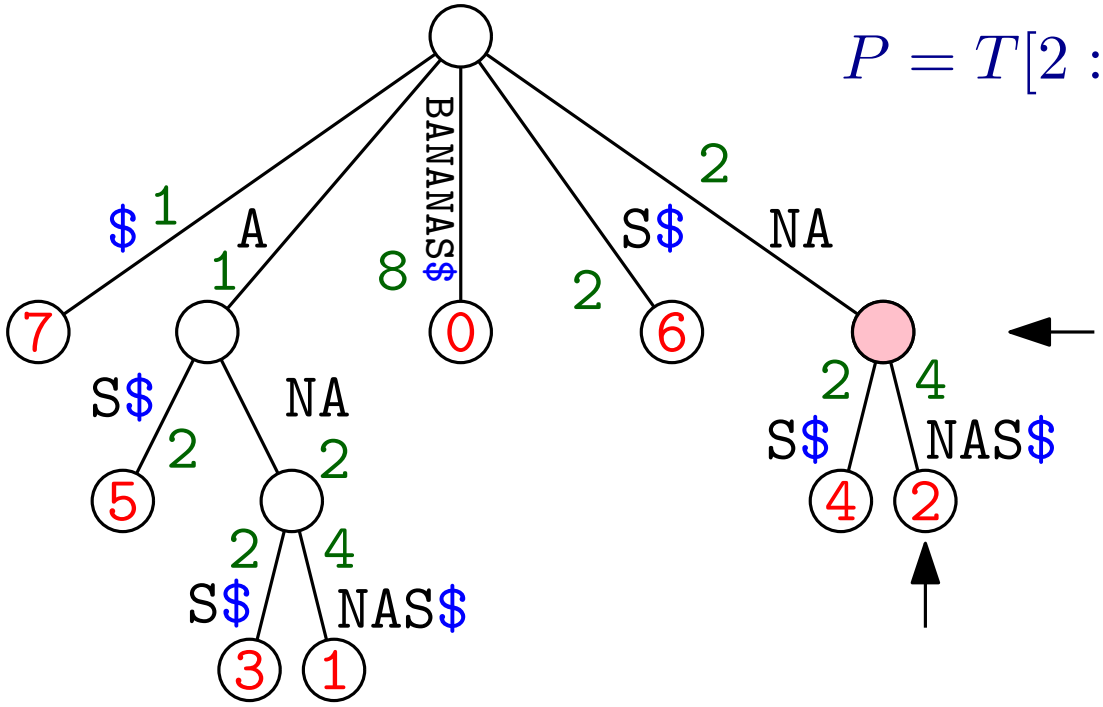

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We can answer weighted LA queries in $O(\log \log |T|)$ time!

Applications: Finding Additional Matches

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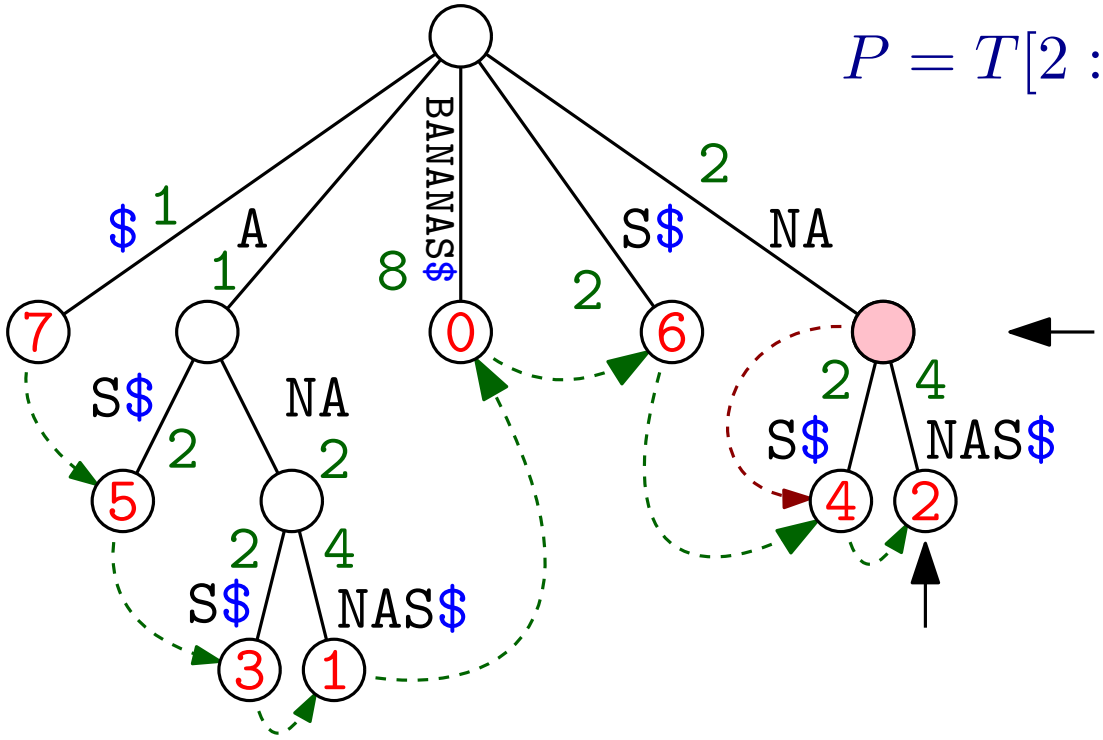

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- We want to quickly find the node that corresponds to P
- Start from the leaf corresponding to $T[i:]$
- Walk **up** the tree for "$|T| - j$" characters
- This is a **weighted** level ancestor query!
- Link leaves to find the other occurrences in $O(1)$ additional time each

We can answer weighted LA queries in $O(\log\log|T|)$ time!

Applications: Document Retrieval

Preprocess collection of documents $T_1, T_2, \ldots, T_k$ to quickly find all documents that contain a pattern P

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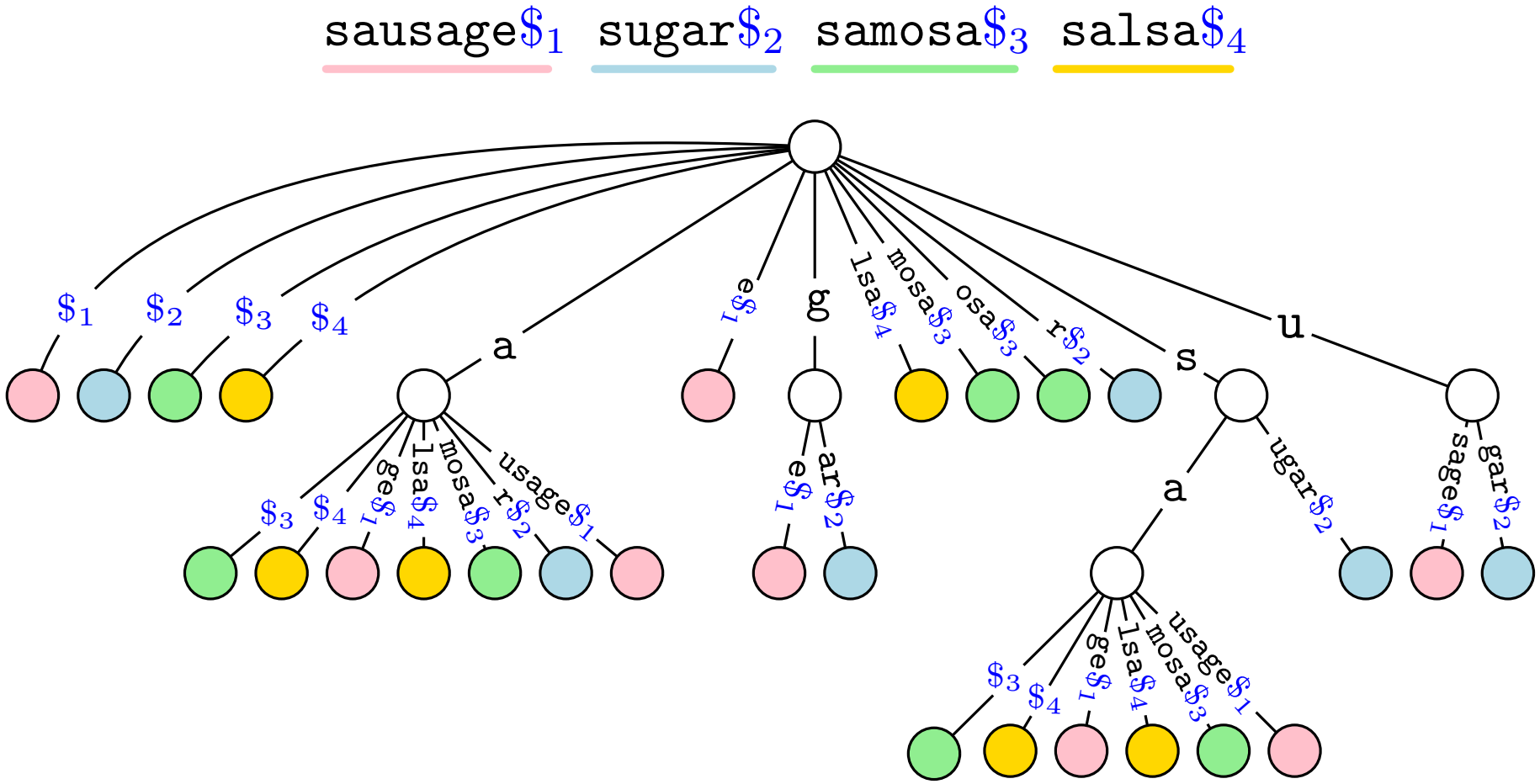
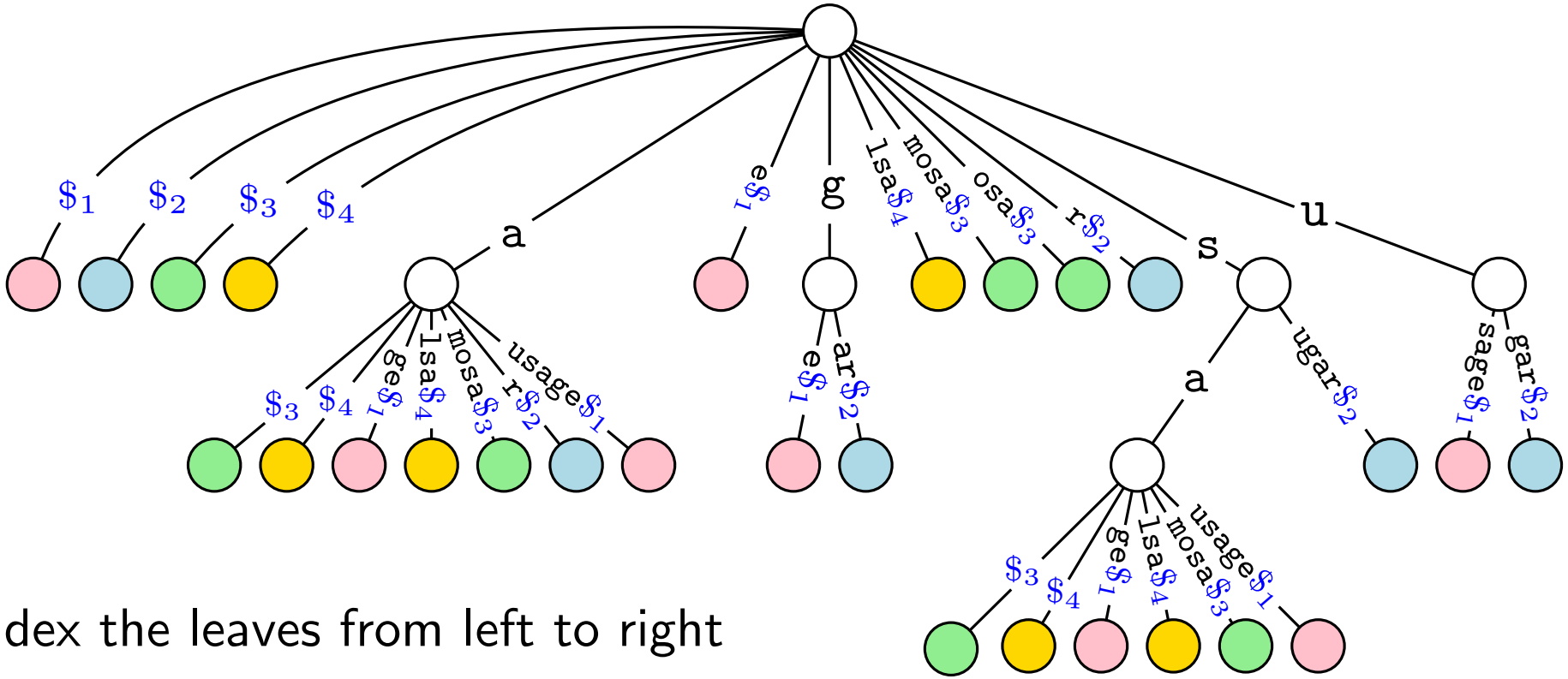
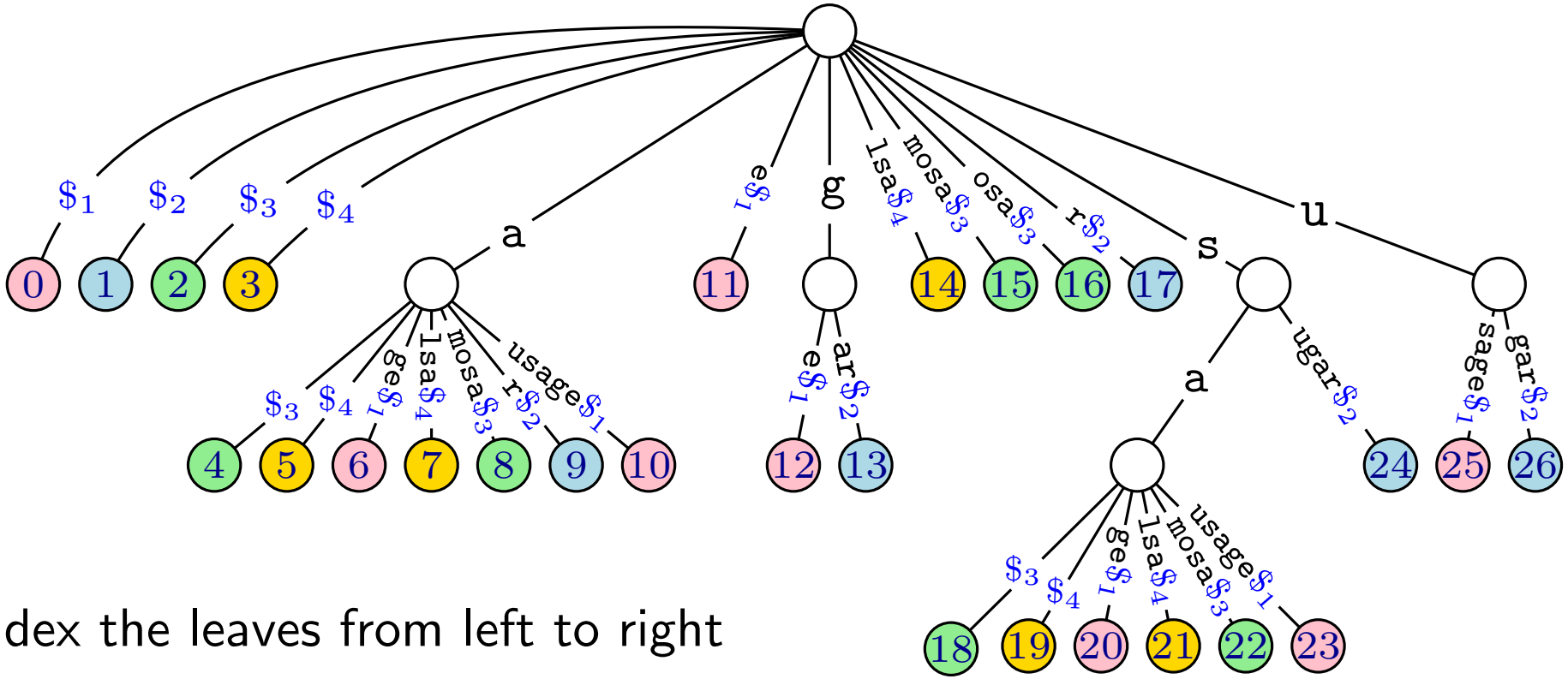


Applications: Document Retrieval



Index the leaves from left to right

Applications: Document Retrieval



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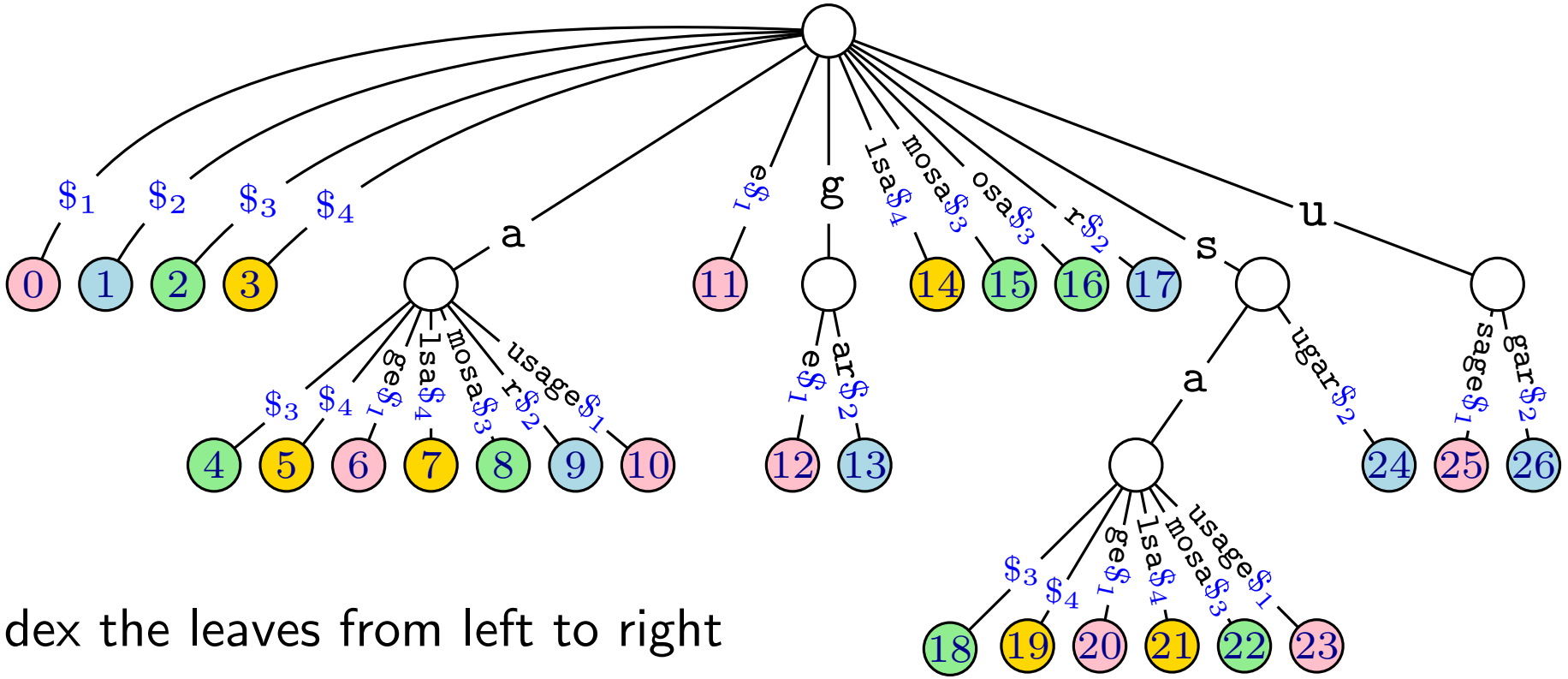
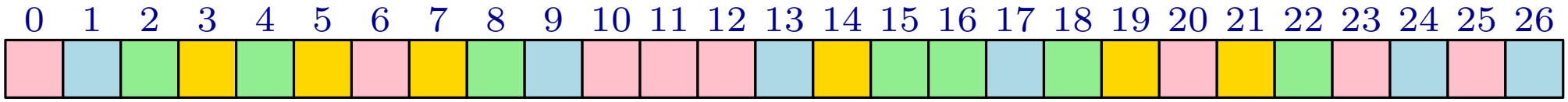
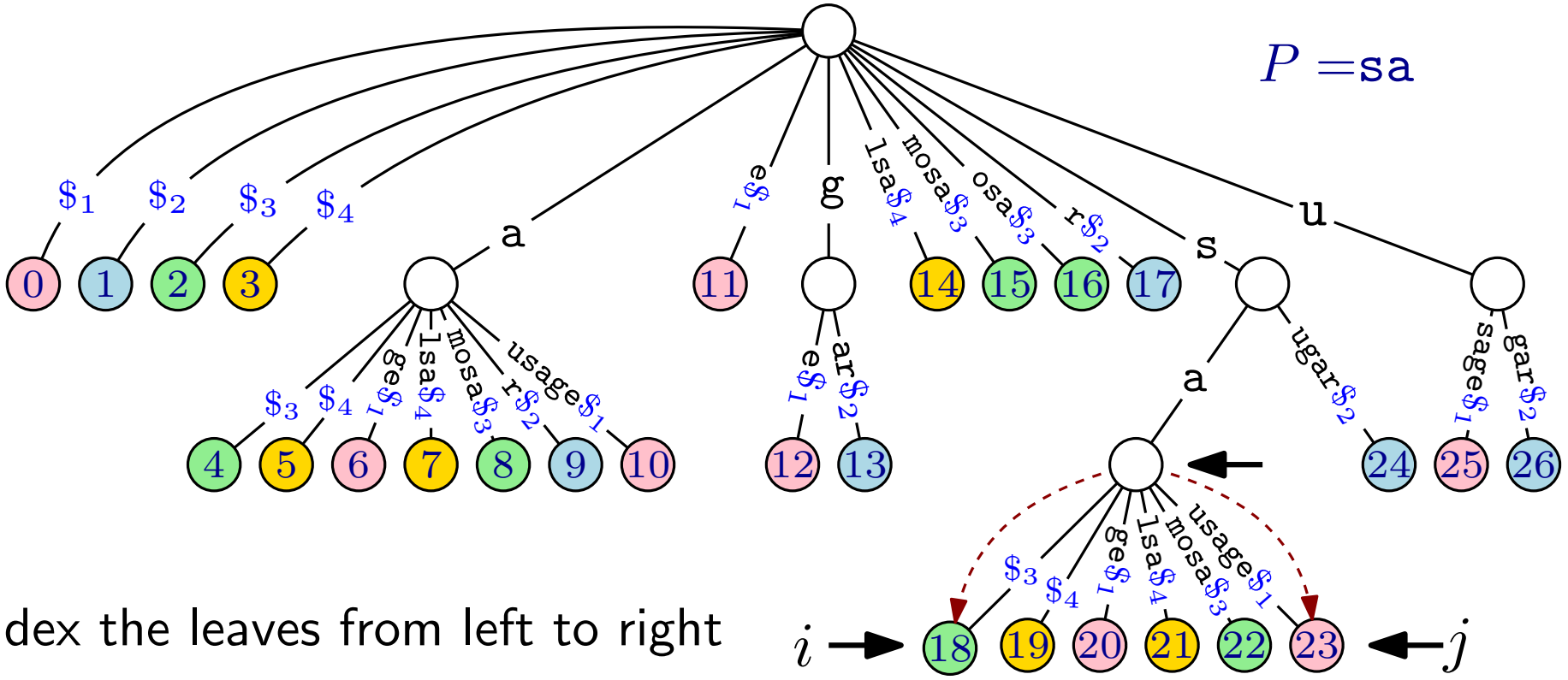


Index the leaves from left to right

Store an array A where $A[i]$ poinst to the document of leaf i



Applications: Document Retrieval



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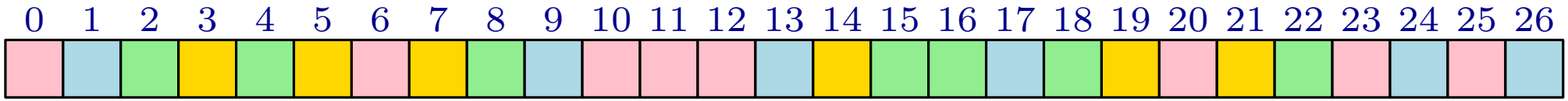

Searching for a pattern P returns the interval $A[i:j]$ containing all and only the leaves corresponding to the matches of P

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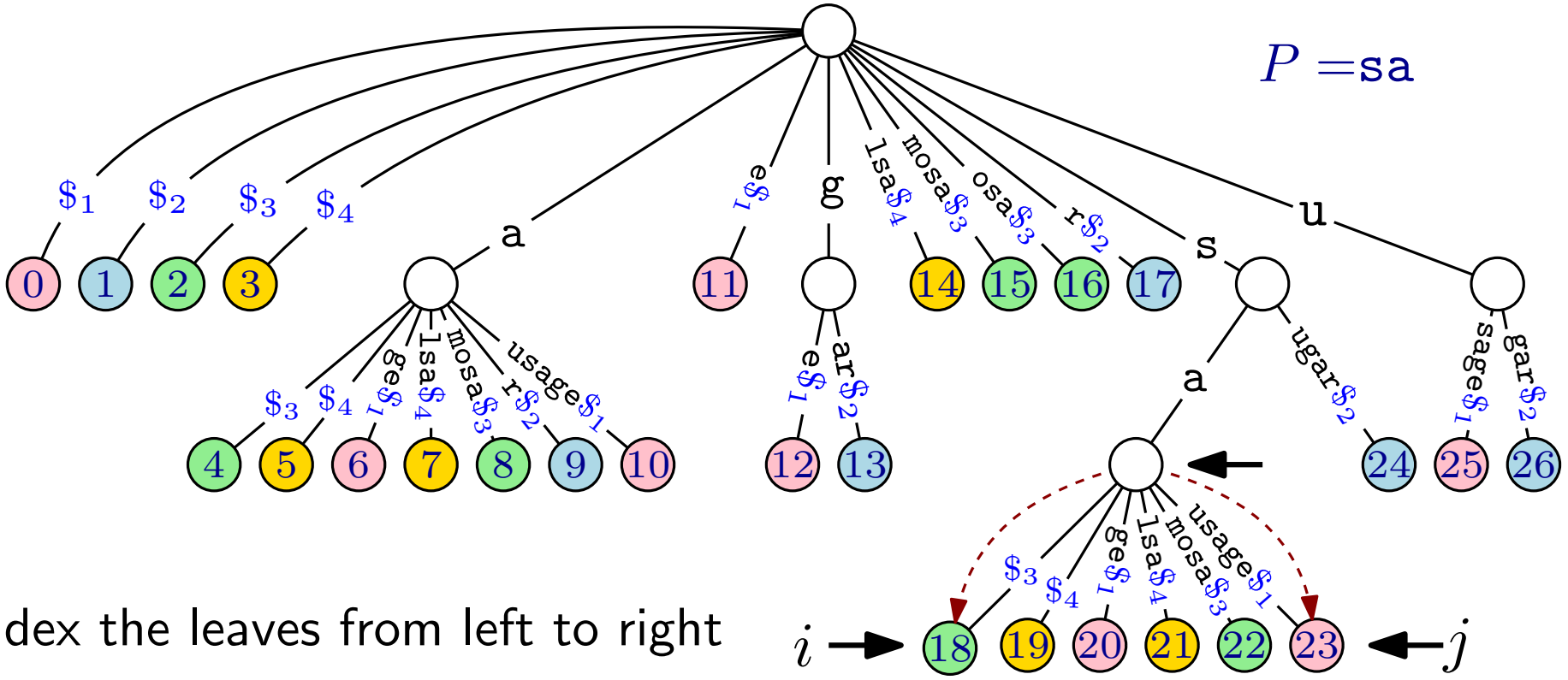

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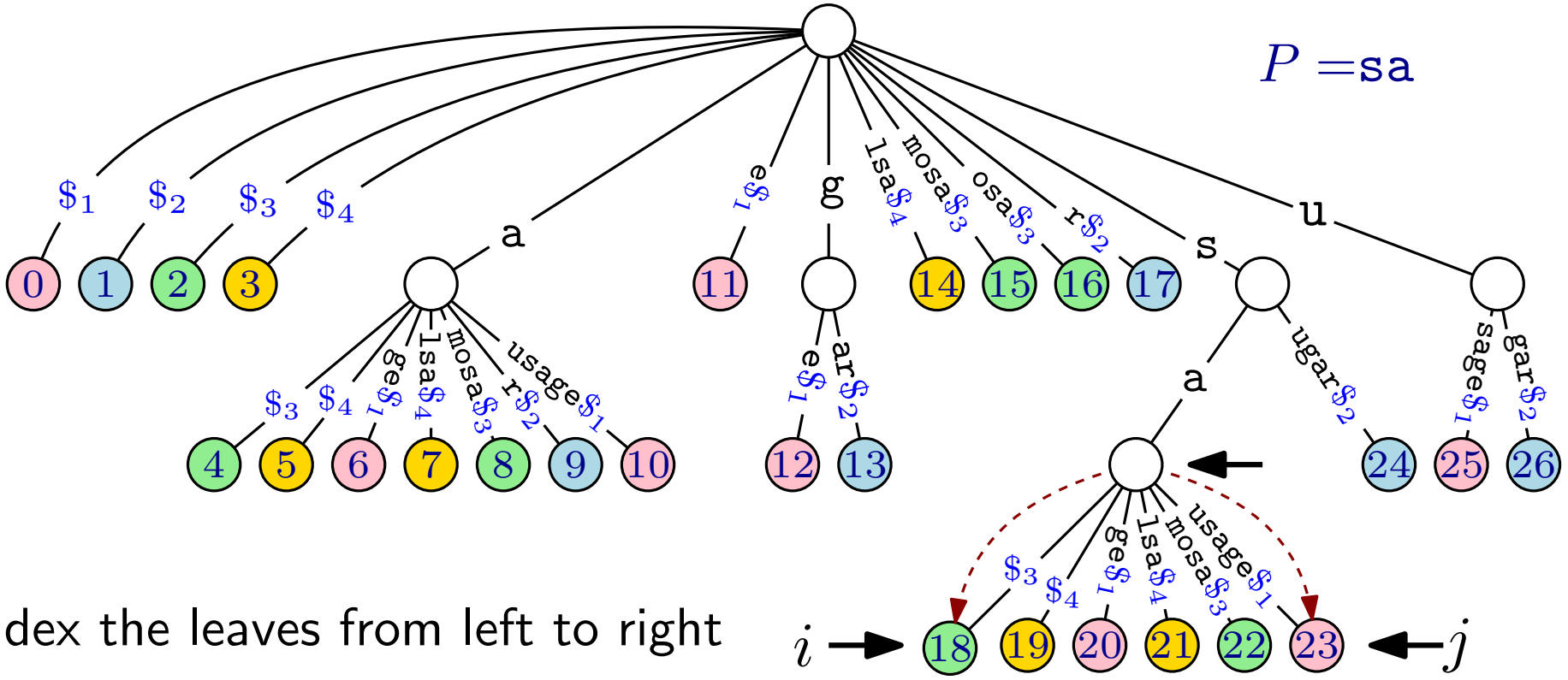

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Searching for a pattern P retu
only the leaves corresponding

Find all distinct documents (c

Time:
$O(|P| + \log|\Sigma| + \#$ retrieved documents)
via range minimum queries

Constructing Suffix Trees
&
Suffix Arrays

Suffix Arrays & Suffix Trees

$T =$ BANANAS

01234567

Sort all suffixes along with their start index

0 BANANAS$
1 ANANAS$
2 NANAS$
3 ANAS$
4 NAS$
5 AS$
6 S$
7 $

Suffix Arrays & Suffix Trees

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Suffix array

Suffix Arrays & Suffix Trees

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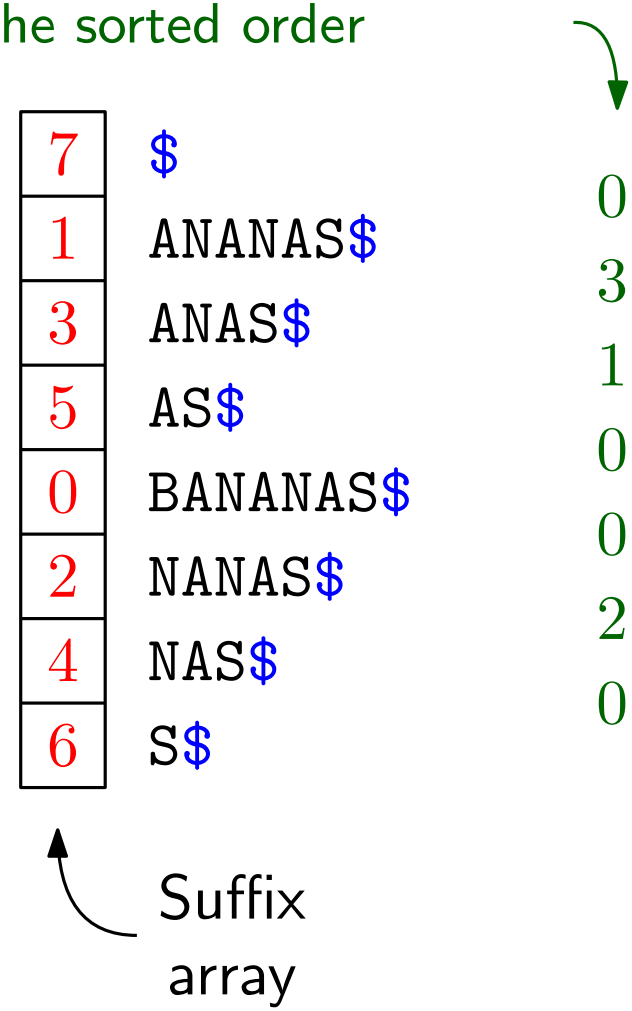
$T =$ BANANAS

Length of the longest common prefix between adjacent suffixes in the sorted order



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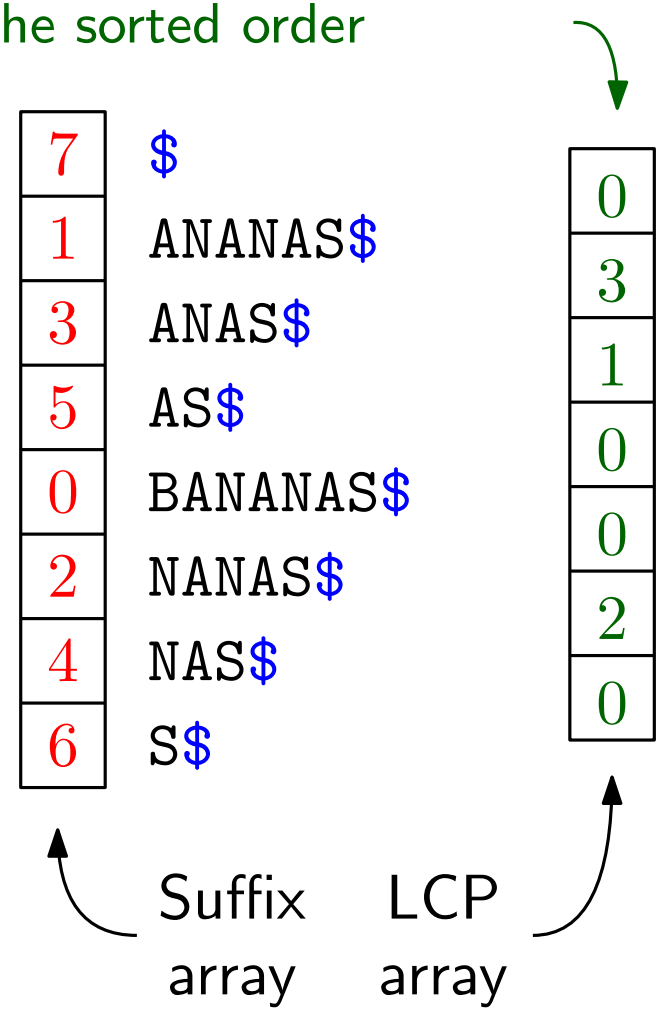

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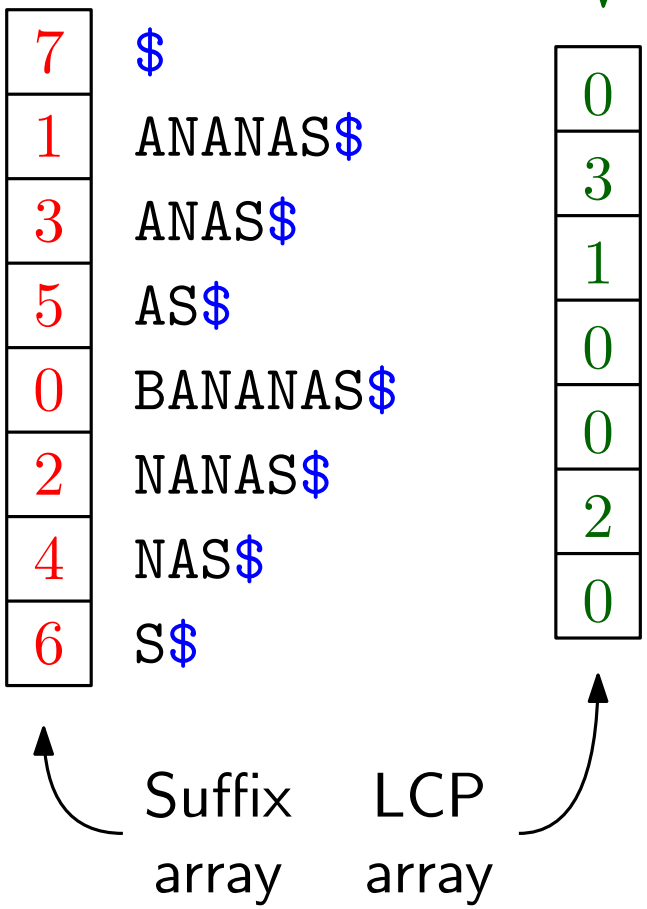

We can construct a suffix tree from the suffix and LCP arrays

A construction similar to the one of cartesian trees yields the subtree of branching vertices

0	3	1	0	0	2	0

Suffix Arrays & Suffix Trees

01234567

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Length of the longest common prefix between adjacent suffixes in the sorted order

7	$	0
1	ANANAS$	3
3	ANAS$	1
5	AS$	0
0	BANANAS$	0
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Suffix array

LCP array

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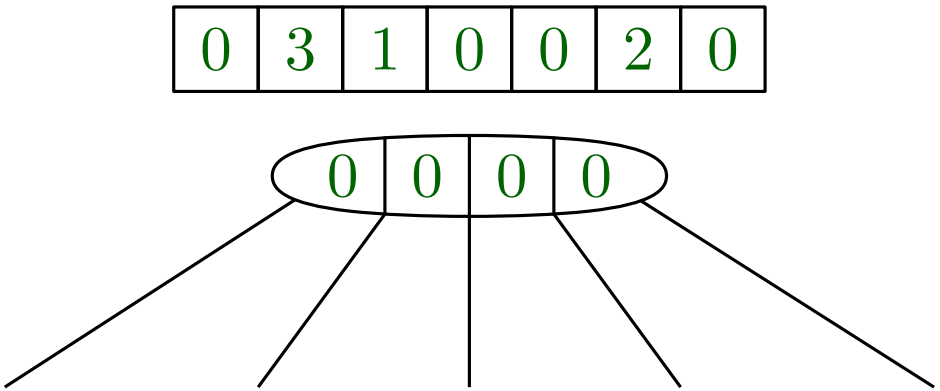

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Suffix array

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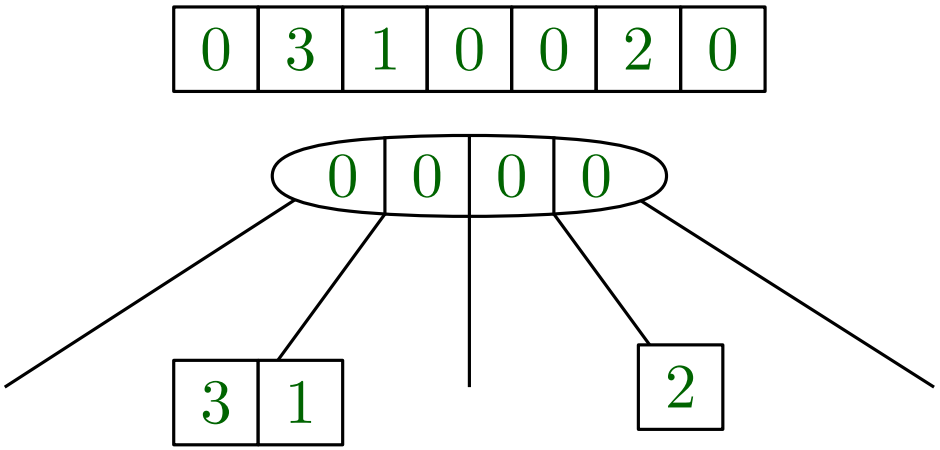

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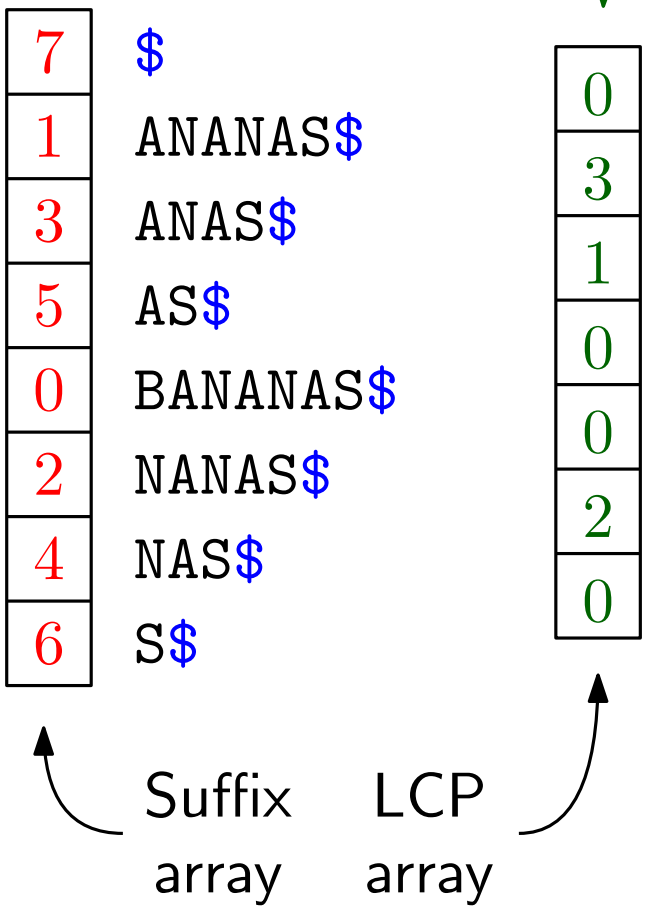

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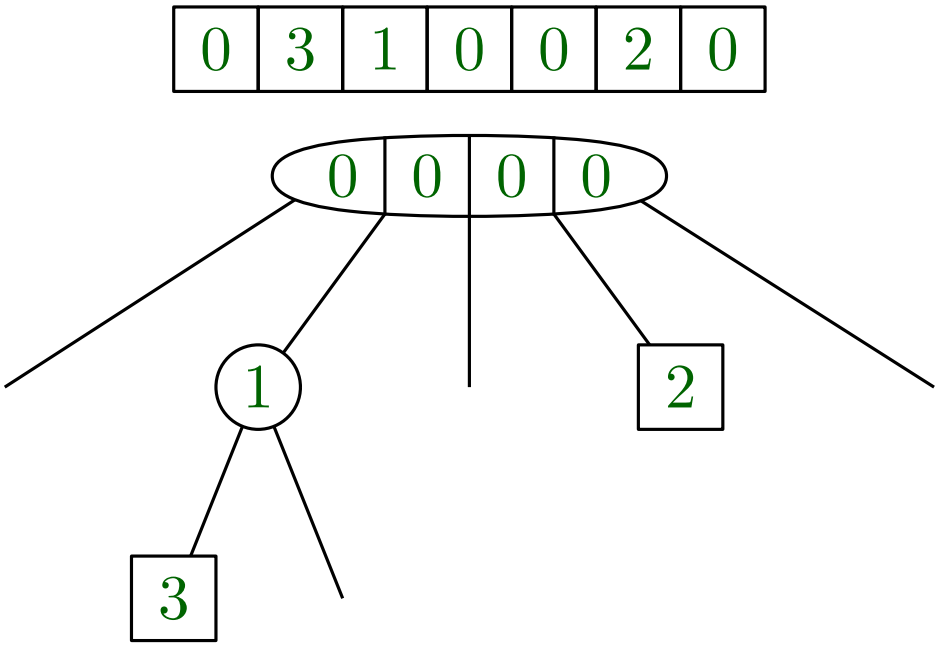

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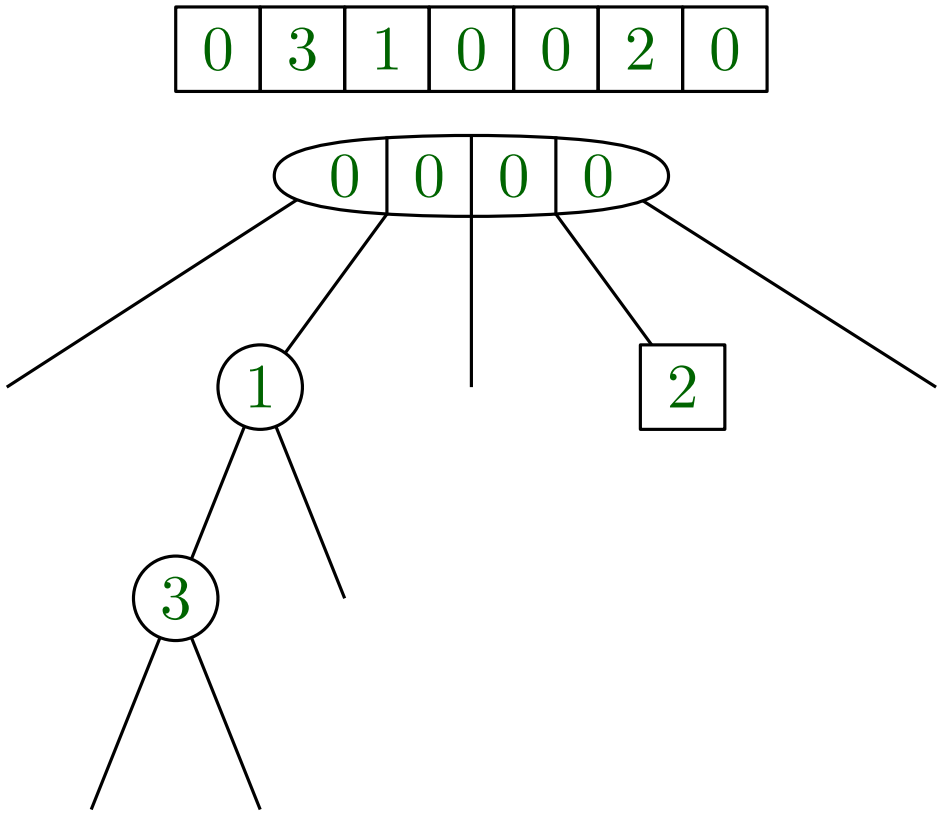

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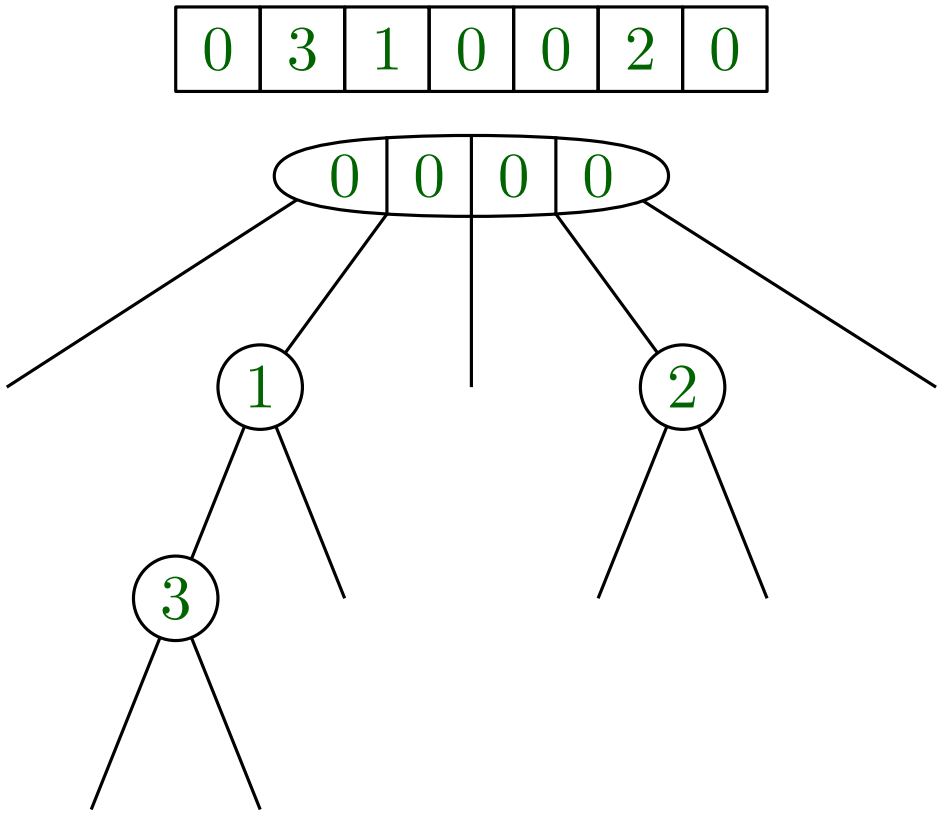

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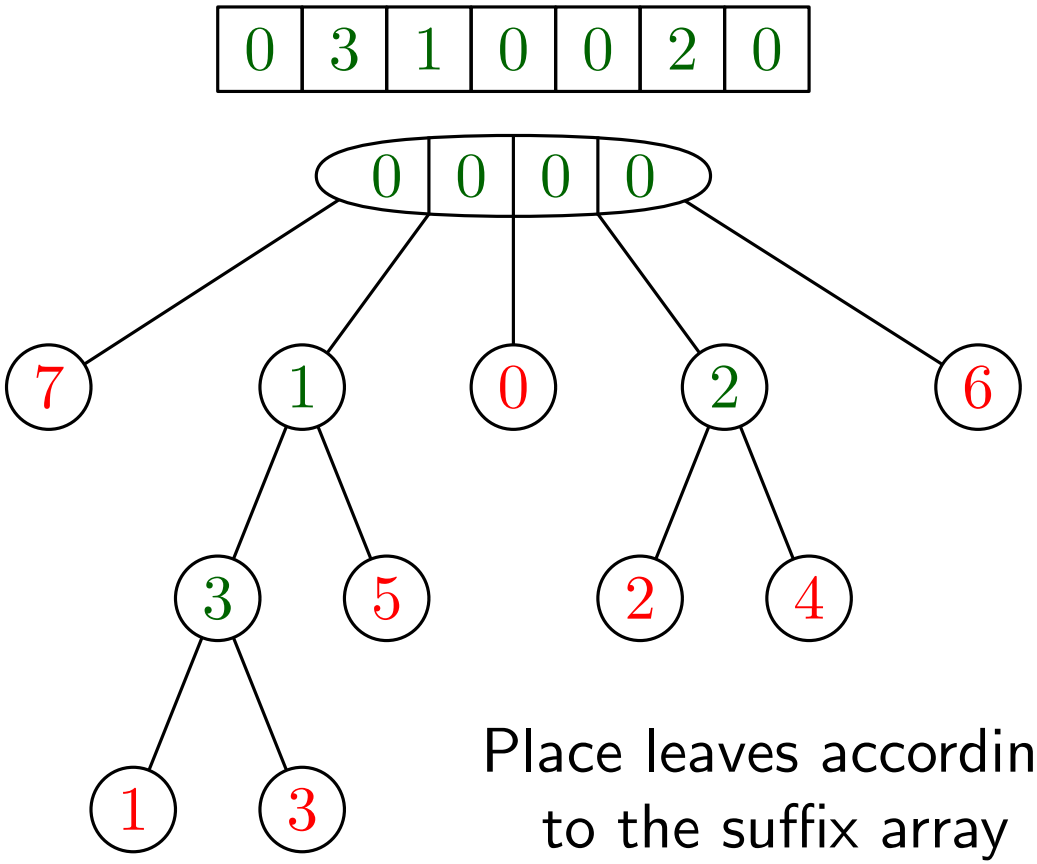
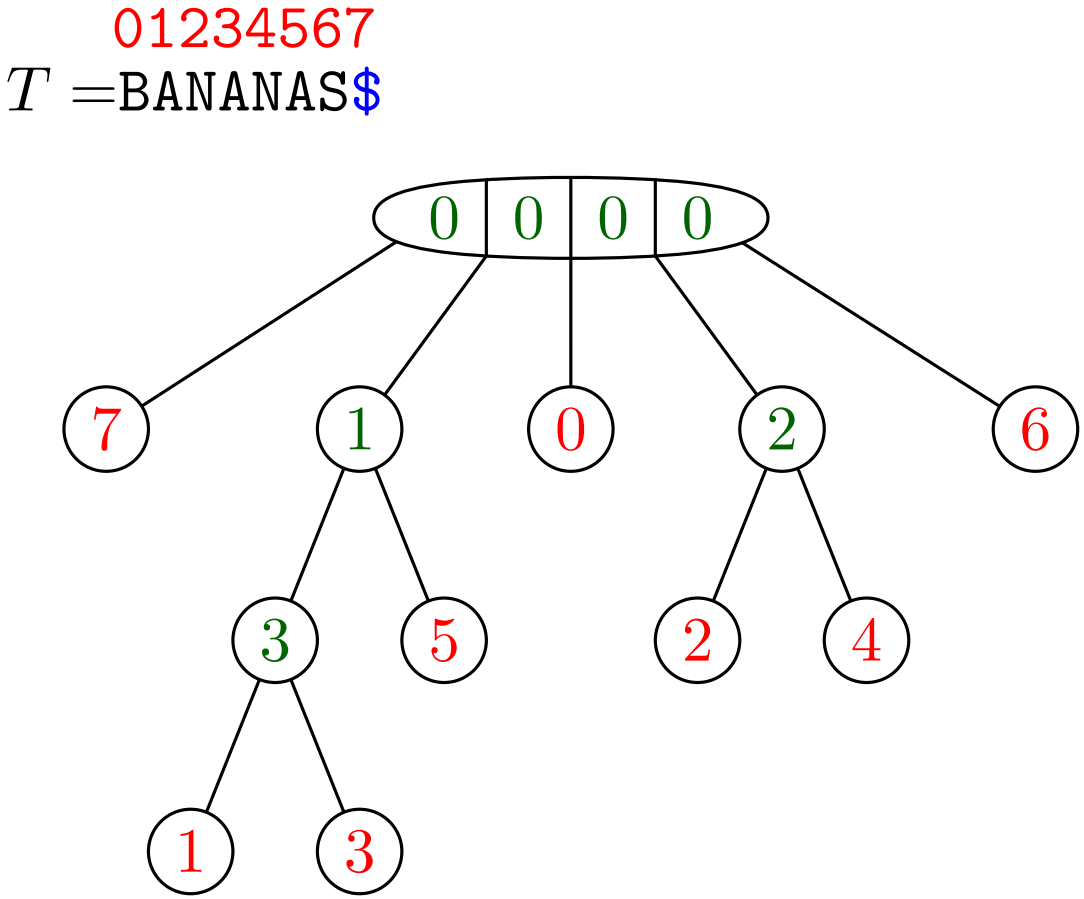


Place leaves according to the suffix array

Suffix Arrays & Suffix Trees

Branching vertices are labelled with their *letter depth*, i.e., the number of letters of the prefix encoded in the path from the root to the vertex



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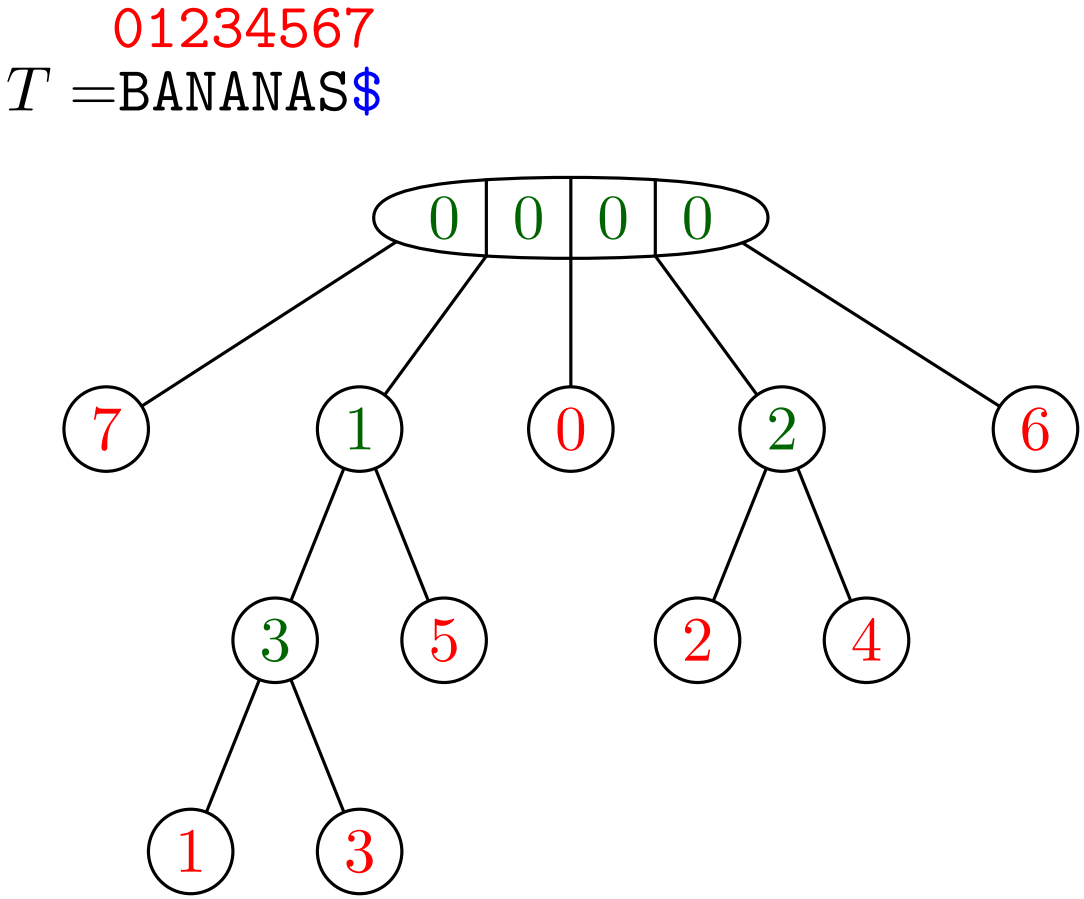

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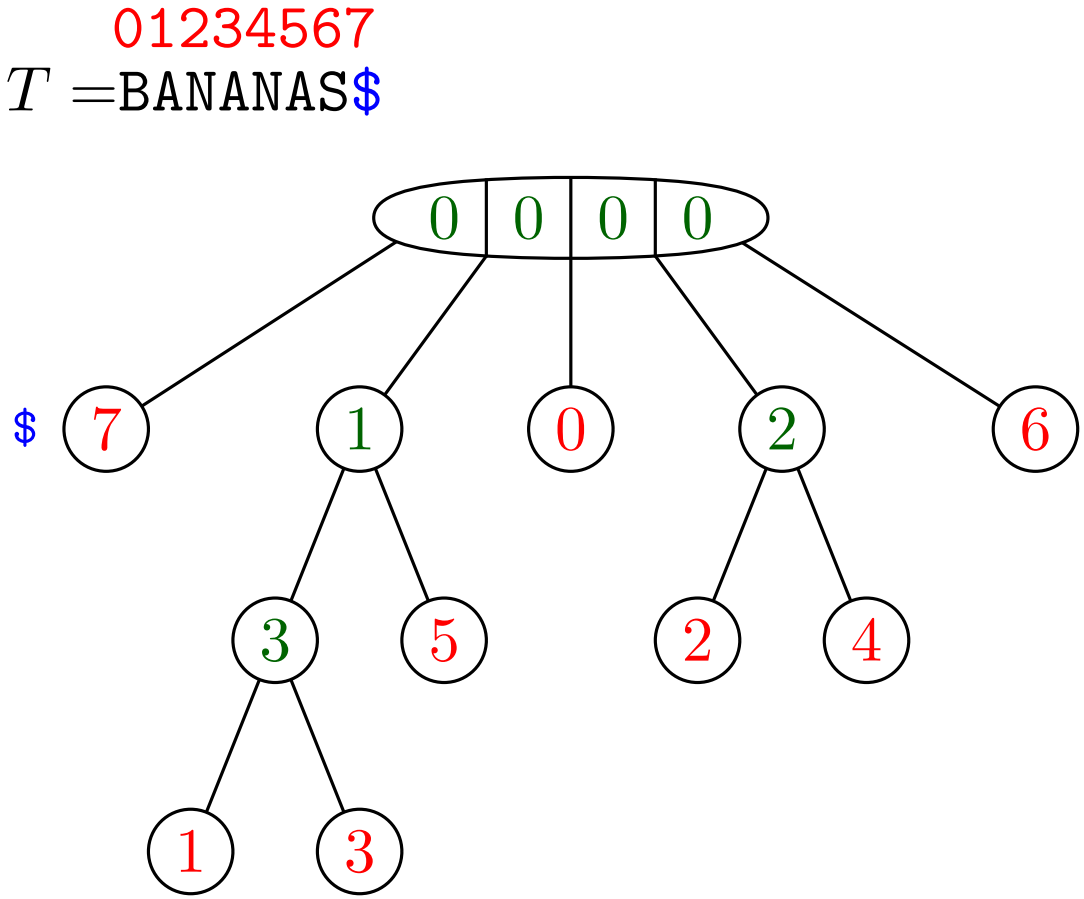

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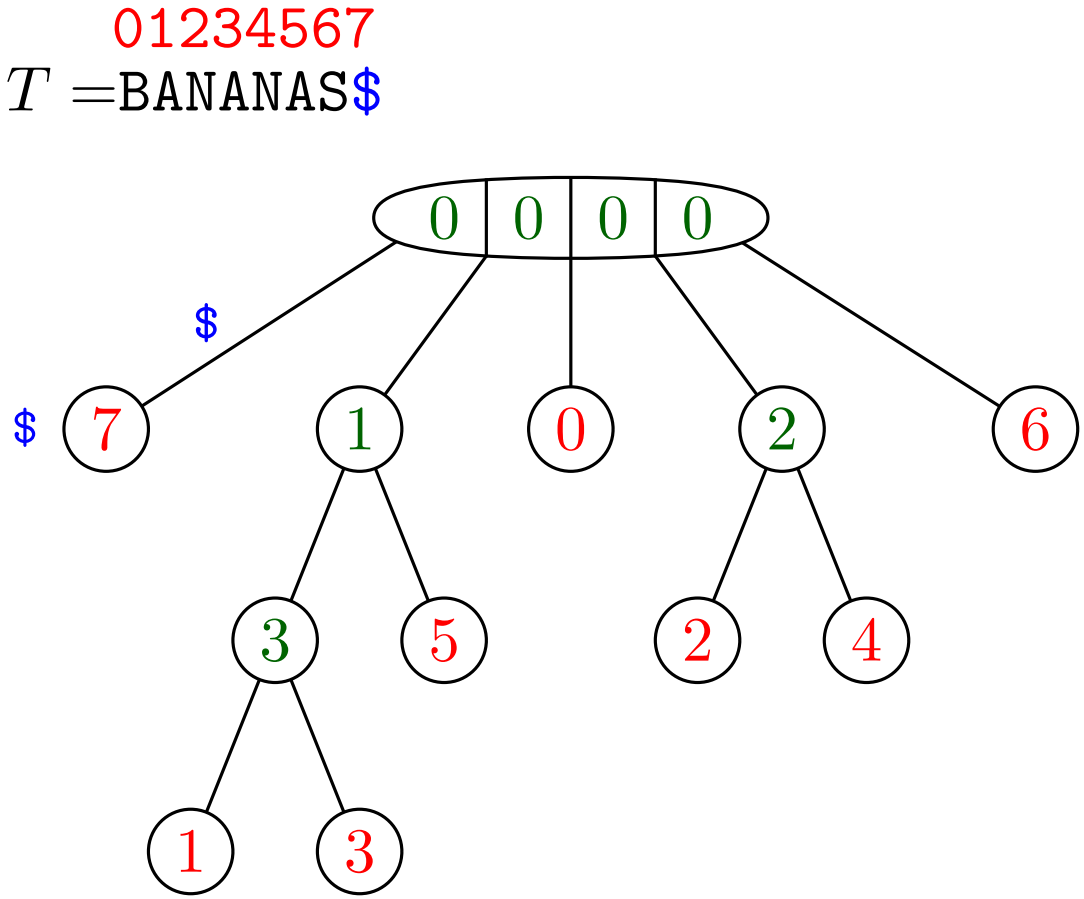

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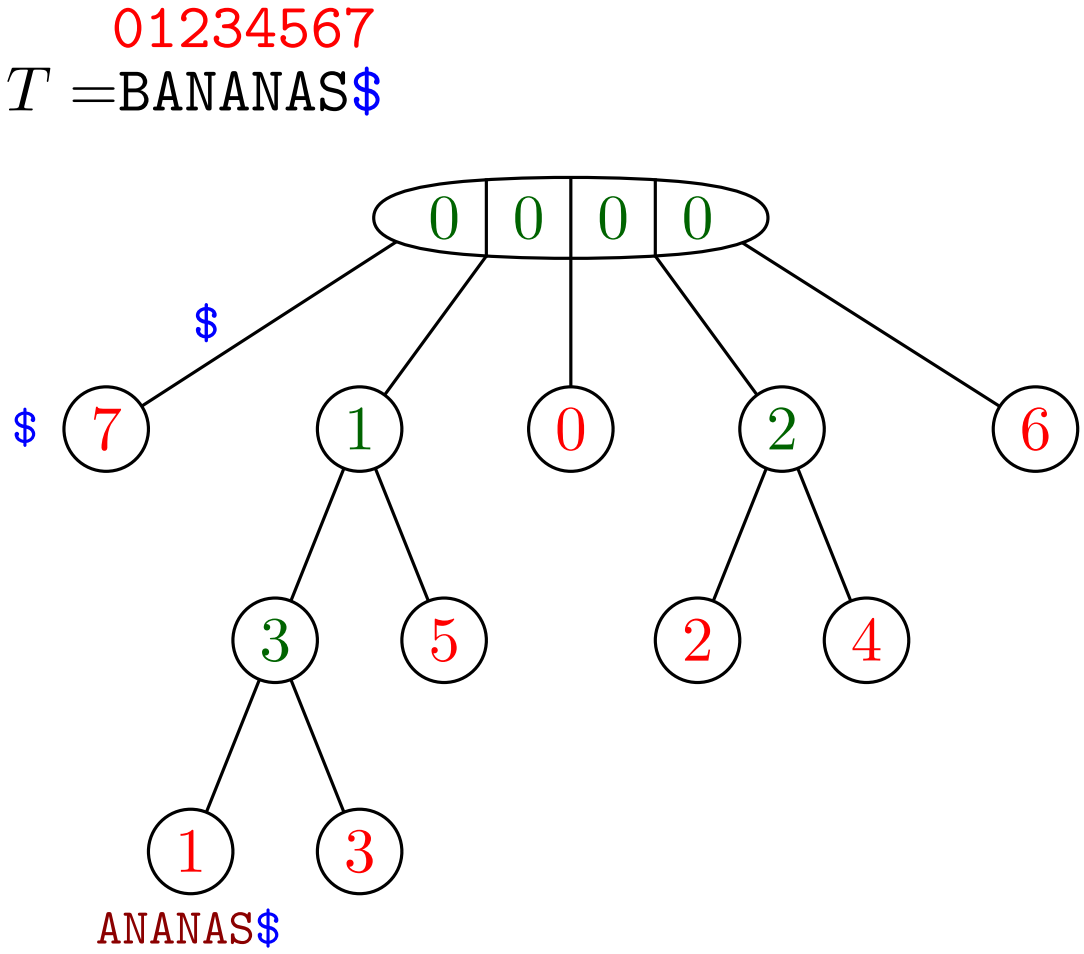

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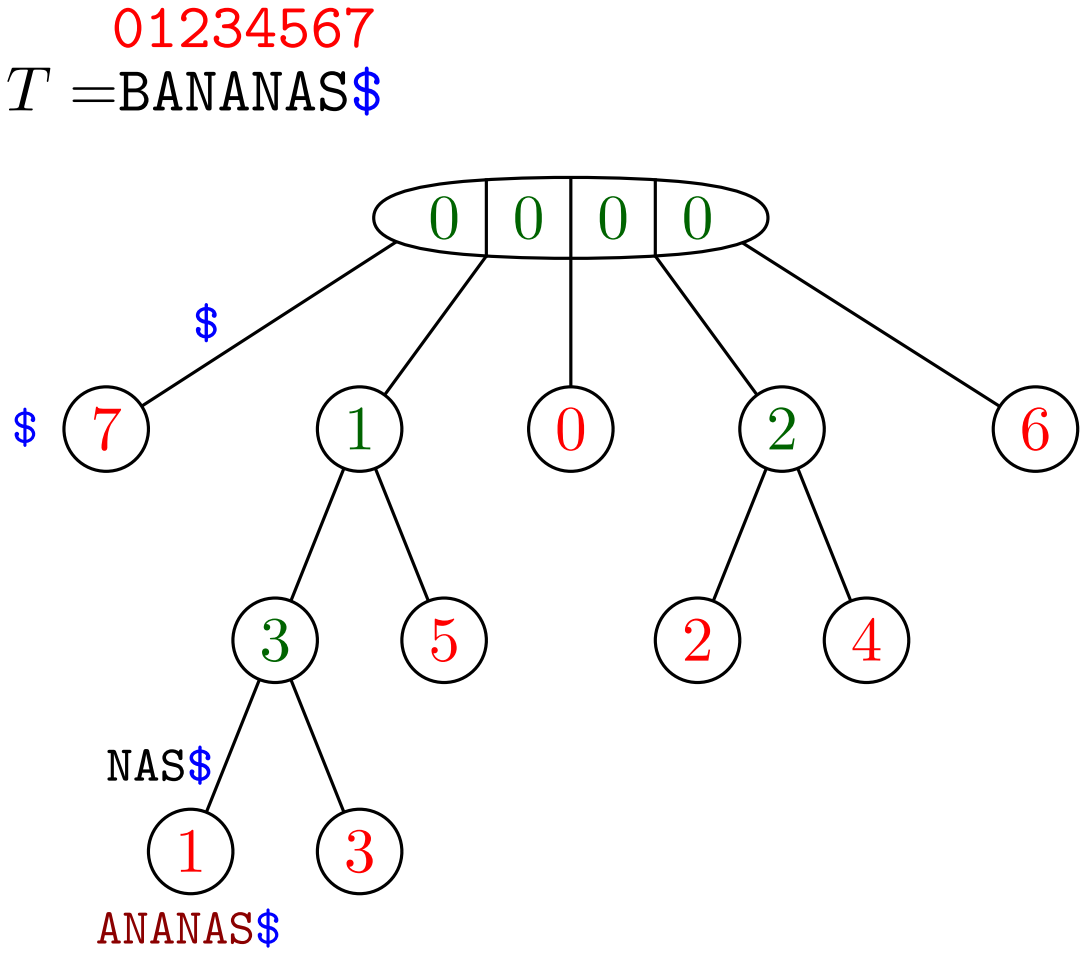

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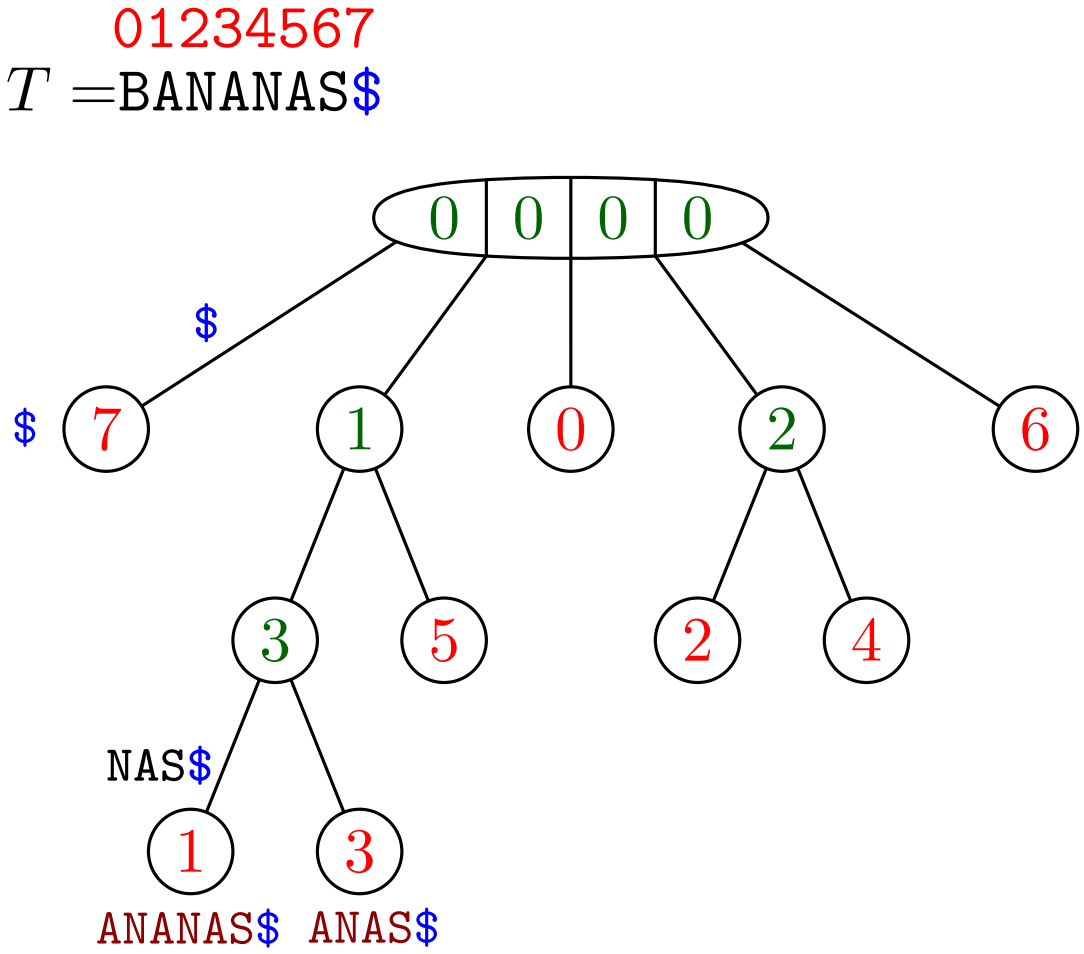

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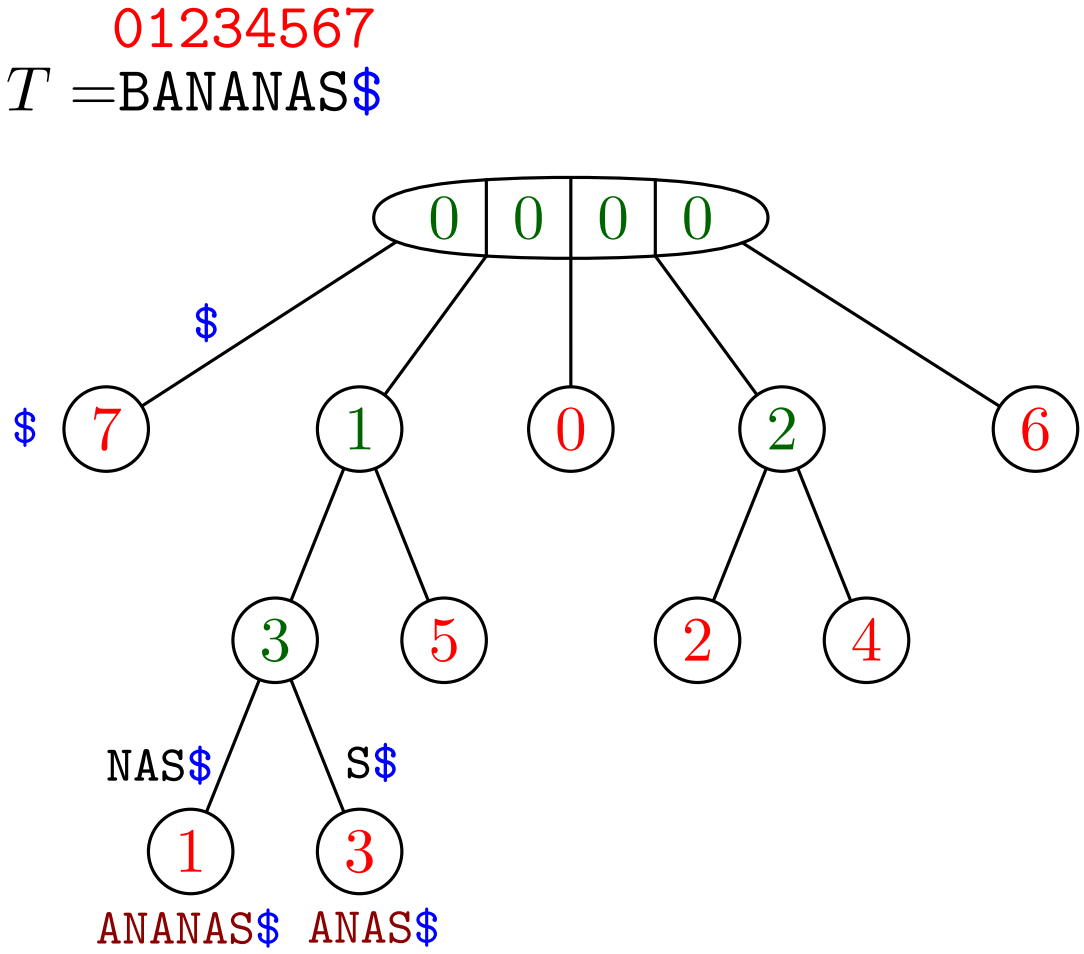

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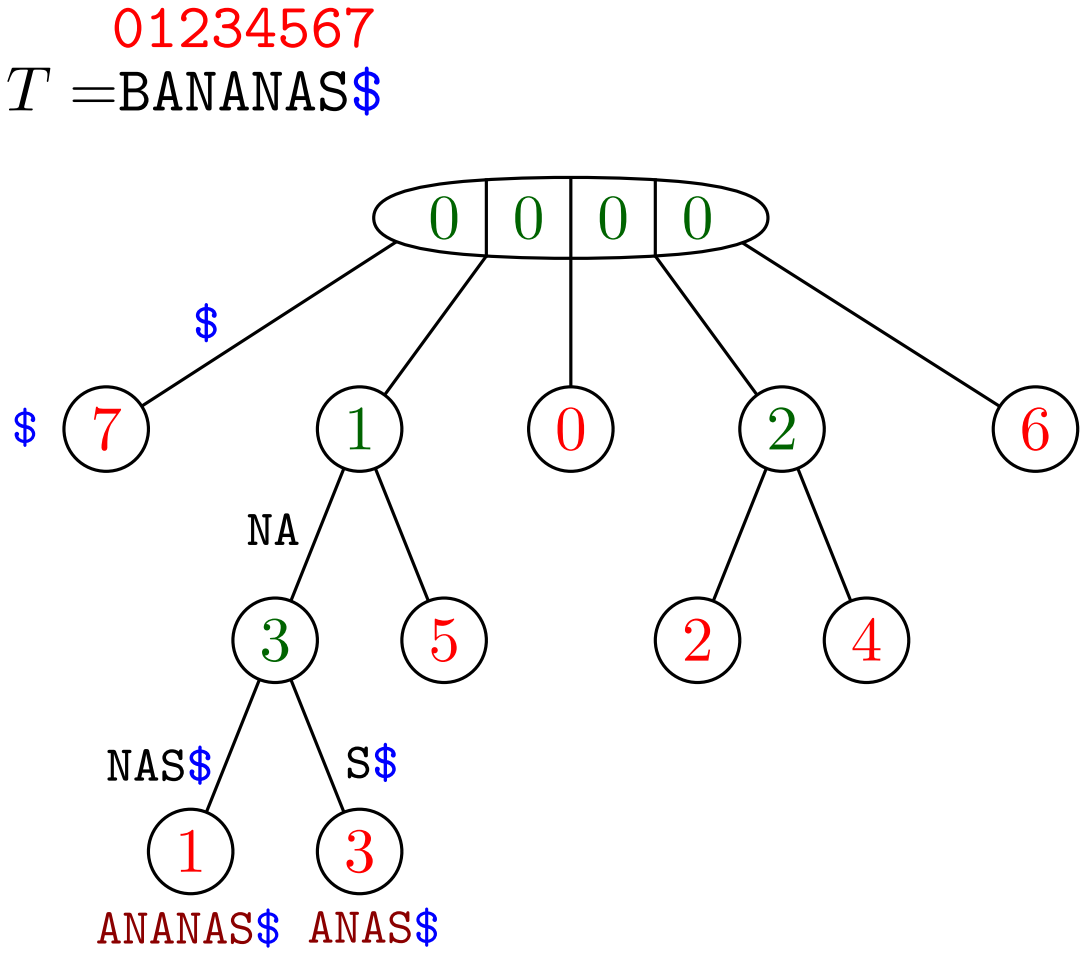

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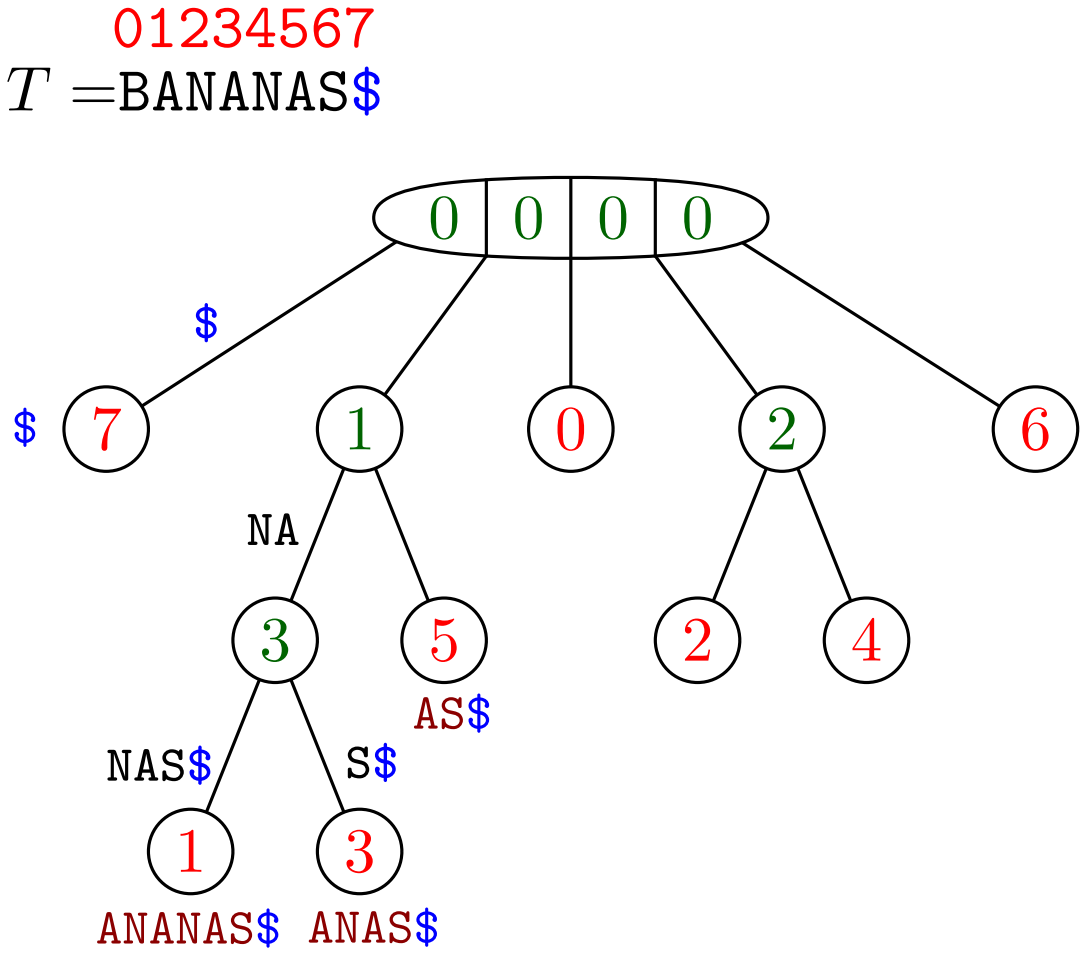

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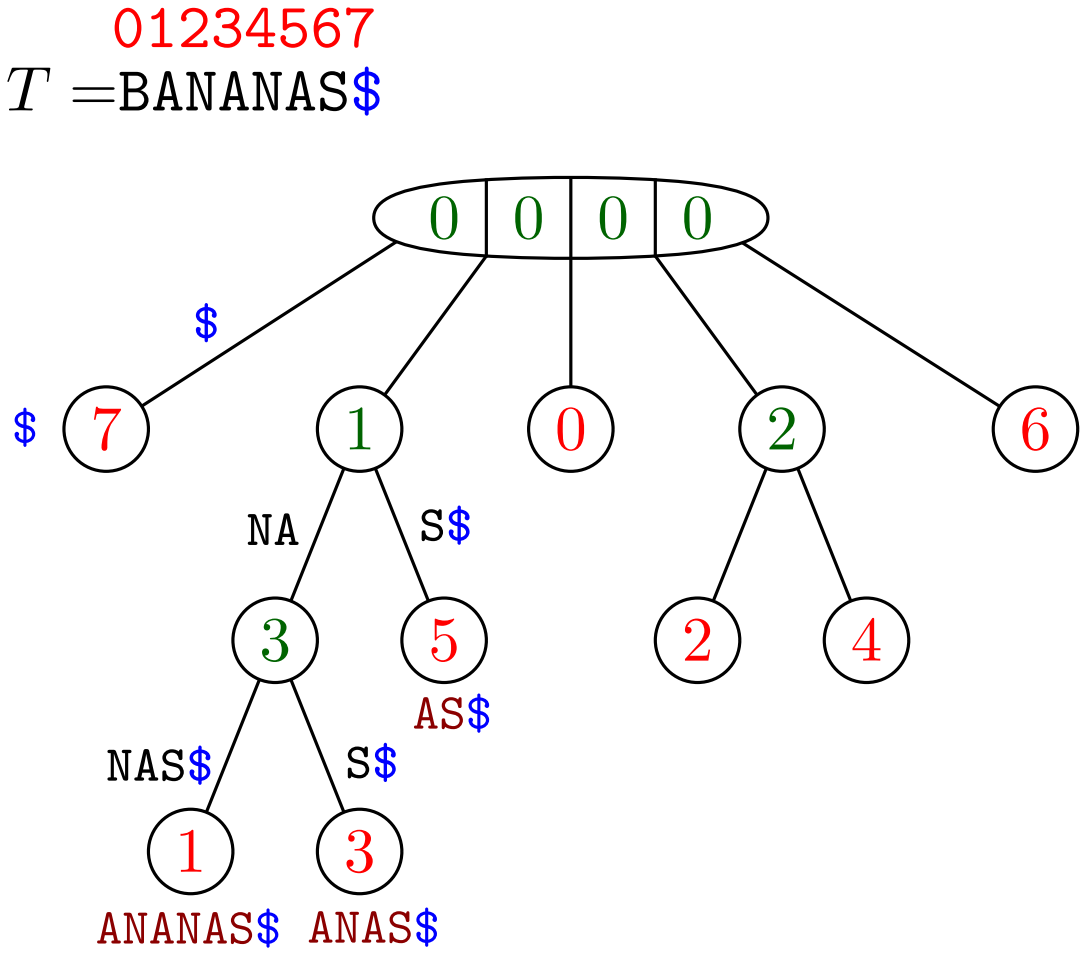

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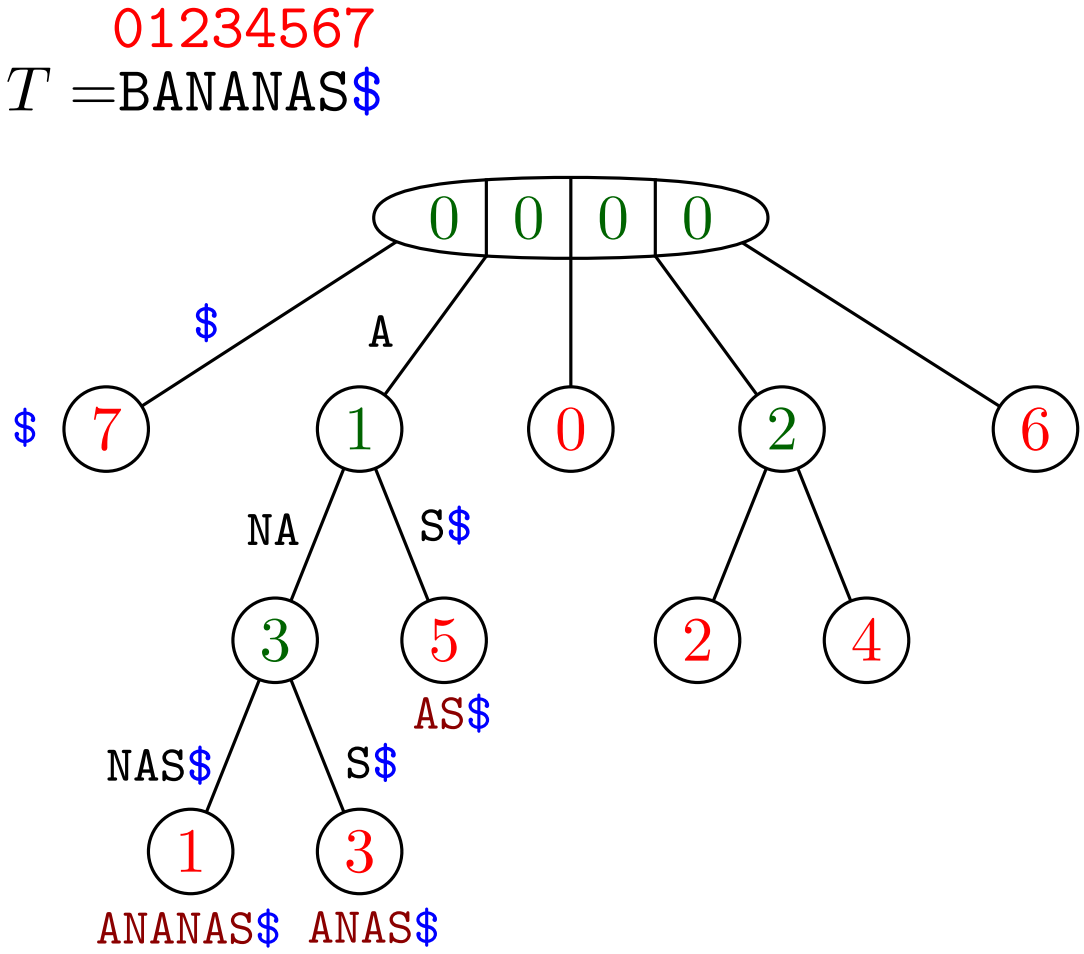

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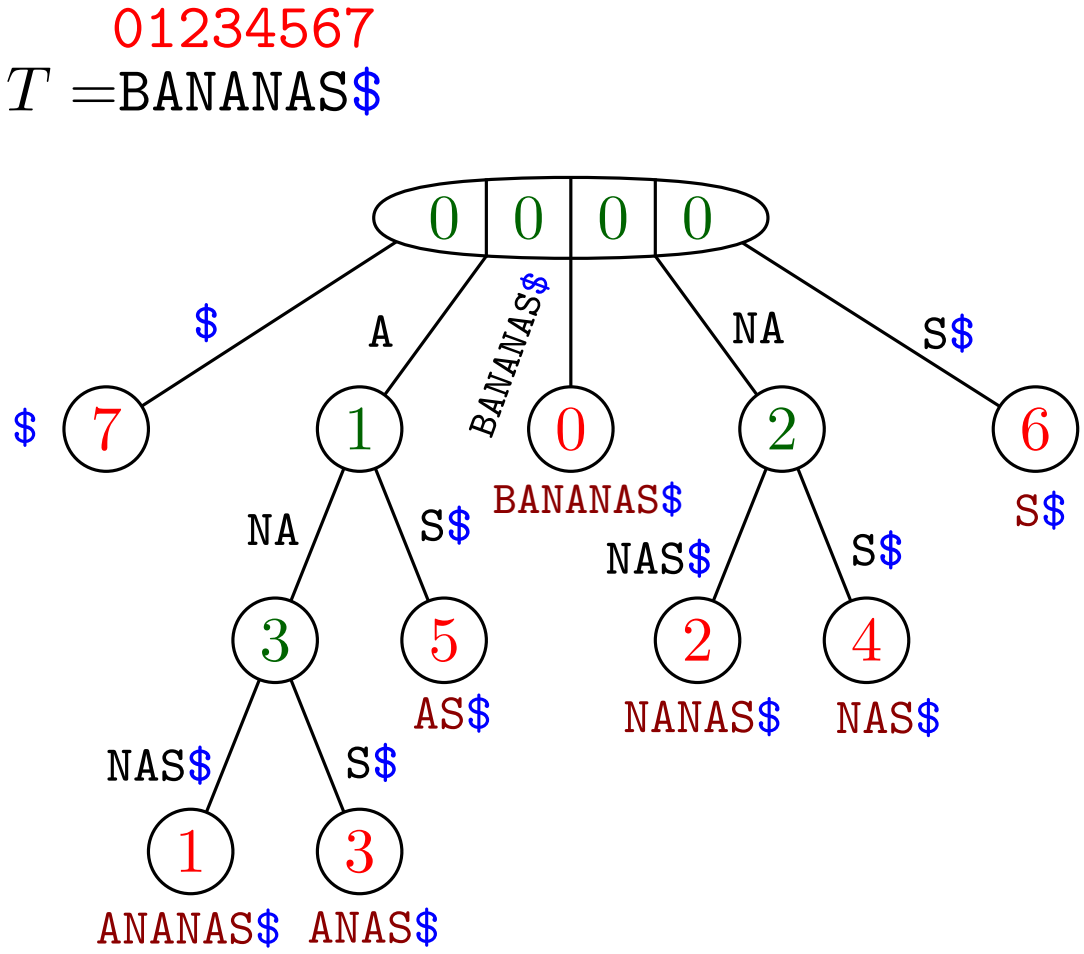

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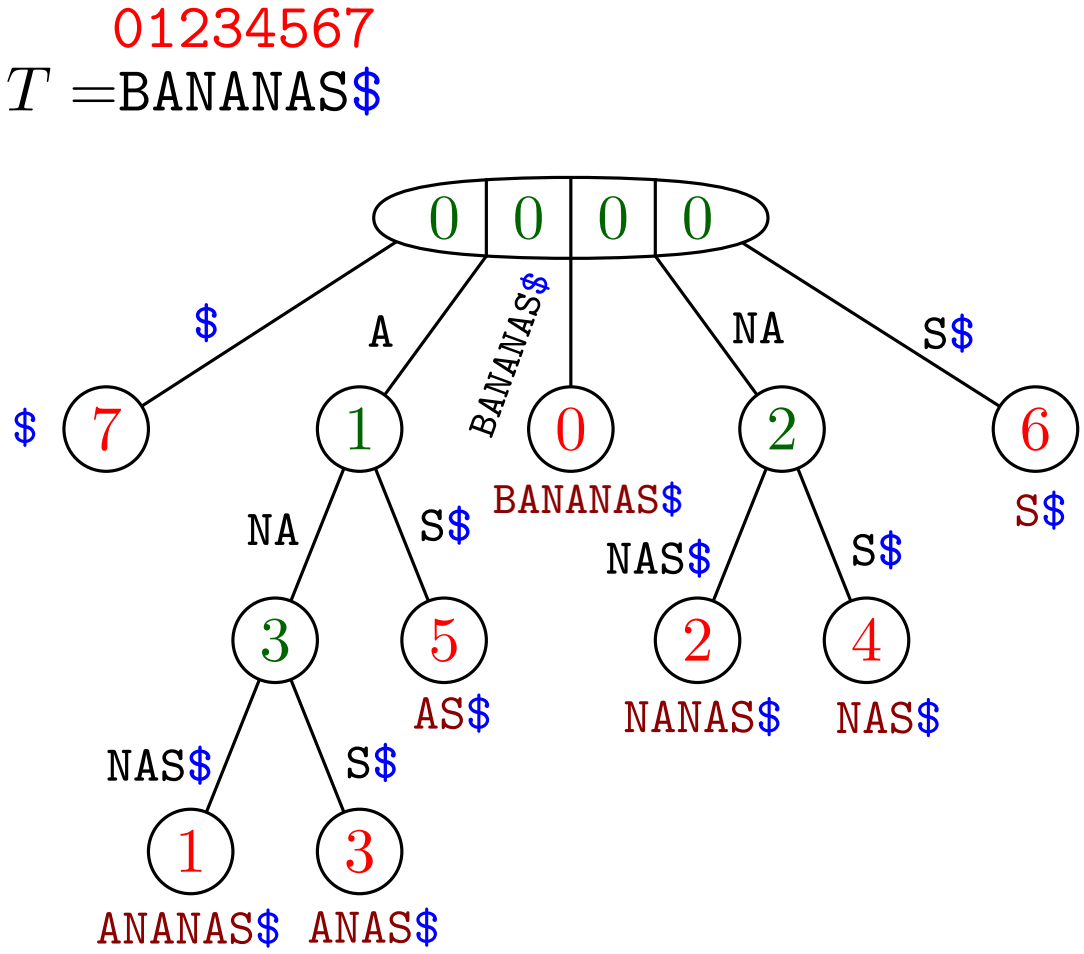

Construction time
(given Suffix + LCP Arrays):
$O(|T|)$

Suffix Arrays & Suffix Trees

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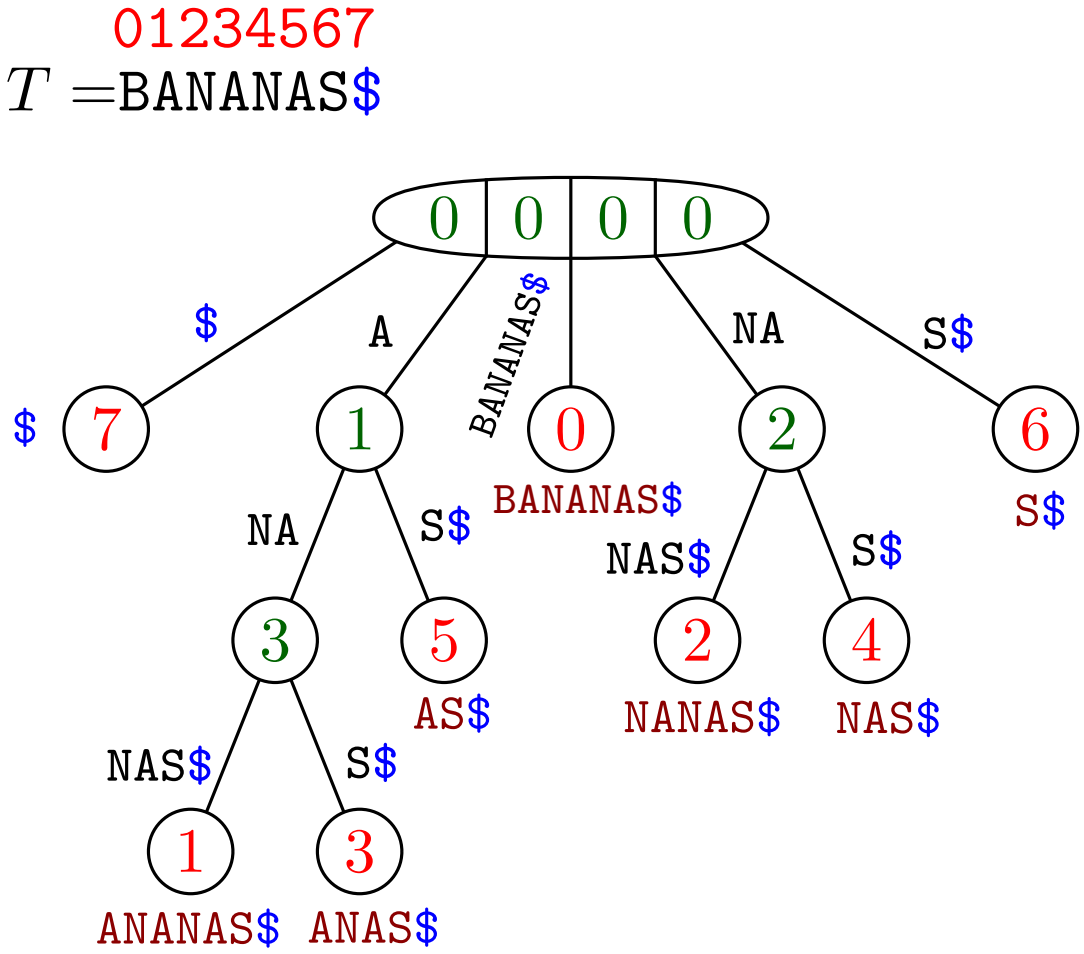

Construction time (given Suffix + LCP Arrays): $O(|T|)$

Suffix + LCP Arrays can be built in $O(|T|)$ time

[J. Kärkkäinen, P. Sanders, ICALP'03]

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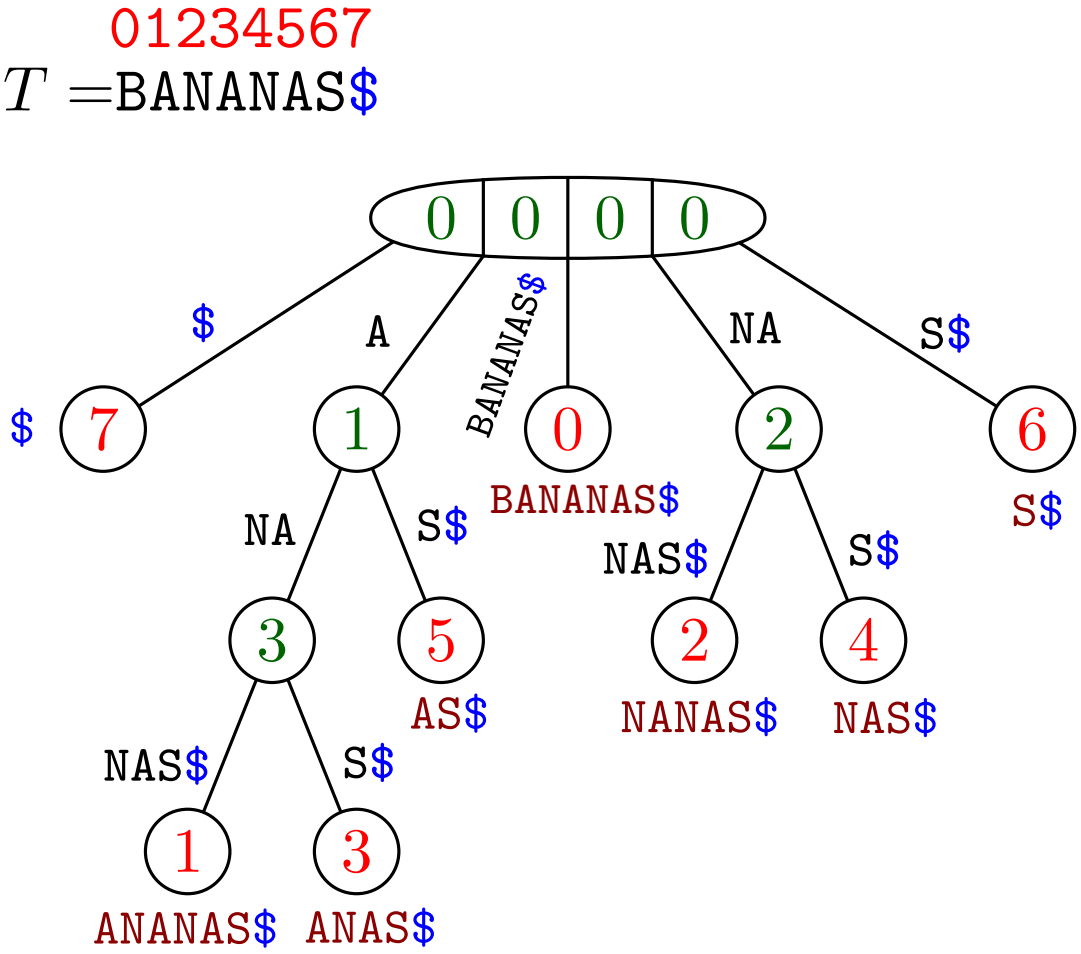

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⇓

Suffix trees can be built in $O(|T|)$ time!