Algorithm Design Laboratory with Applications

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Problem: *Number station.*

A number station is a radio station that broadcasts sequences of numbers for espionages operations. You are a secret agent and you just received a transmission with the combination of a safe where vital intelligence is held. The transmission is encrypted and consists of a $n \times n$ matrix $A = (a_i, j)_{i,j}$ of non-negative integers. As per your instructions, you are to recover the combination by counting the number of *non-empty contiguous submatrices* of A whose sum of elements is odd.

Time is key: design and implement an algorithm to *quickly* compute the safe combination.

Input. The input consists of a set of instances, or *test-cases*, of the previous problem. The first line of the input contains the number T of test-cases. The first line of each test-case is the integer n. The i-th of the following n lines describes the i-th row of A and contains the n integers $a_{i,1}, \ldots, a_{i,n}$ separated by a space.

Output. The output consists of T lines, each containing a single integer. The i-th line is the answer to the i-th test-case and is the number of quadruples of integers (h, k, x, y) with $1 \leq h \leq k \leq n$ and $1 \leq x \leq y \leq n$ such that $\sum_{i=h}^{k} \sum_{j=x}^{y} a_{i,j}$ is odd.

Example. The safe combination associated with the matrix

$$A = \begin{bmatrix} 1 & 6 & 5 \\ 1 & 4 & 8 \\ 0 & 2 & 4 \end{bmatrix}$$

is 16, since A has 16 non-empty contiguous submatrices whose elements sum to an odd number. Namely: $\begin{bmatrix}1\end{bmatrix}$, $\begin{bmatrix}5\end{bmatrix}$, $\begin{bmatrix}1\end{bmatrix}$, $\begin{bmatrix}1 & 6\end{bmatrix}$, $\begin{bmatrix}6 & 5\end{bmatrix}$, $\begin{bmatrix}1 & 4\end{bmatrix}$, $\begin{bmatrix}1 & 4 & 8\end{bmatrix}$, $\begin{bmatrix}5\\8\end{bmatrix}$, $\begin{bmatrix}1\\0\end{bmatrix}$, $\begin{bmatrix}5\\8\\4\end{bmatrix}$, $\begin{bmatrix}6 & 5\\4 & 8\end{bmatrix}$, $\begin{bmatrix}1 & 4\\0 & 2\end{bmatrix}$, $\begin{bmatrix}1 & 6 & 5\\1 & 4 & 8\end{bmatrix}$, $\begin{bmatrix}1 & 4 & 8\\0 & 2 & 4\end{bmatrix}$, $\begin{bmatrix}6 & 5\\4 & 8\\2 & 4\end{bmatrix}$, and A itself. Notice that the submatrix $\begin{bmatrix}1\end{bmatrix}$ is counted twice since it appears two times in A.

Input (corresponding to matrix A in the above example):

```
1
3
1 6 5
1 4 8
0 2 4
```

Output:

```
16
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Assumptions. $1 \leq T \leq 10$; $1 \leq n \leq 512$; $\forall i, j = 1 \ldots, n,\ 0 \leq a_{i,j} \leq 1024$.

Requirements. Your algorithm must have an asymptotic time complexity of $O(n^3)$.

Notes. A reasonable implementation should not require more than 1 second for each input file.