Algorithm Design Laboratory with Applications

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Problem: *Travelling salesman.*

A long highway traverses n cities, indexed from 1 to n. ACME Inc., a large multinational company, owns several warehouses in some of these cities.

A salesman of ACME is planning a business trip: he will start his trip from a warehouse in a city i, where he will collect the products to sell, and he will end the trip in a city j with $j \geq i$, where he will drop the unsold products in a warehouse. The salesman will sell the products in each of the cities encountered during the trip: when he encounters a city $k \in [i, j]$ he expects to sell goods for s_k dollars. The salesman also knows that travelling from a generic k-th city to the next city $k+1$ costs c_k. This means that if he chooses i and j as the endpoints of his trip, his expected earnings will be of $E(i,j) = \sum_{k=i}^{j} s_k - \sum_{k=i}^{j-1} c_k$ dollars.

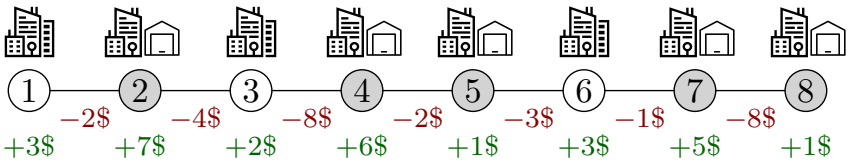
Design an algorithm that, given (i) the locations of the cities, (ii) the set W of the (indexes of the) cities that contain ACME warehouses, (iii) the costs c_k, and (iv) the expected amount s_k of goods sold in the generic k-th city, finds the best possible trip for the salesman, i.e., computes two indices $i, j \in W$ with $j \geq i$ such that $E(i,j)$ is maximized.

Input. The input consists of a set of instances, or *test-cases*, of the previous problem. The first line of the input contains the number T of test-cases. The first line of each test-case contains the overall number of cities n and the number $|W|$ of cities with a warehouse. The second line contains the n values $s_1, \ldots, s_n$. The third line contains the $n-1$ values $c_1, \ldots, c_{n-1}$. Finally, the fourth and last line of the test-case contains $|W|$ integers corresponding to the indices of the cities in W, in increasing order.

Output. The output consists of T lines, where the i-th line is the answer to the i-th test-case and contains $\max_{\substack{i,j \in W \\ i<j}} E(i,j)$.

Assumptions. $1 \leq T \leq 10$; $1 \leq n < 2^{21}$; $0 < s_i, c_i < 2^{10}$; $2 \leq |W| \leq n$.

Example.



Input:

```
1
8 5
3 7 2 6 1 3 5 1
2 4 8 2 3 1 8
2 4 5 7 8
```

Output (corresponding to the trip from the 4-th to the 7-th city):

```
9
```

Requirements. Your algorithm should require $O(n)$ time (with reasonable hidden constants).

Notes. A reasonable implementation should not require more than 1 second for each input file.