# Algorithm Design Laboratory with Applications

Prof. Stefano Leucci

## Problem: Tunnel.

A mountain lies between two valleys, one on its west side and the other on its east side. The people of the area decide to build a tunnel to connect the two valleys. The candidate area for excavation can be described as a rectangular  $h \times \ell$  matrix M in which each entry (i, j) represents a square area. Due to the different types of rocks, different areas can be easier or harder to mine: in general, the area at coordinates (i, j) requires  $h_{i,j}$  hours to clear.

The tunnel can start from any location on the east side, i.e., from any entry (i, 1) with  $i = 1, \ldots, h$ , and can end in any location on west side, i.e., in any entry  $(i, \ell)$  with  $i = 1, \ldots, h$ . Moreover, for each  $j = 1 \ldots, \ell$  only one entry among  $\{(1, j), \ldots, (h, j)\}$  can be excavated. Finally, the tunnel needs to be continuous, i.e., if (i, j) and (i', j+1) are excavated then  $|i-i'| \leq 1$ . Design an algorithm that finds the tunnel connecting the west valley to the east valley that requires the least amount of hours H to be built.

## Input.

The input consists of a set of instances, or *test-cases*, of the previous problem. The first line contains the number T of test-cases. The first line of each test case contains the integers h and  $\ell$ . The *i*-th of the following h lines of the test-case contains the  $\ell$  integers  $h_{i,1}, \ldots, h_{i,\ell}$ .

## Output.

The output consists of T lines, each corresponding to a test-case. The *i*-th of the lines contains the integer H corresponding to the minimum amount of hours needed to build a tunnel from the west valley to the east valley.

## Example.



Input (corresponding to the above example):

1
4 6
5 4 6 4 2 3
7 2 7 6 1 2
4 6 5 7 8 4
2 5 3 1 9 1
Output:
19

Assumptions.  $1 \le T < 10;$   $1 \le h < 2^{10};$   $1 \le \ell < 2^{13};$   $\forall i = 1, \dots, h, \forall j = 1, \dots, \ell,$  $1 \le h_{i,j} \le 2^{14}.$ 

**Requirements.** Your algorithm must have an asymptotic time complexity of  $O(h \cdot \ell)$  (with reasonable hidden constants).

Notes. A reasonable implementation should not require more than 1 second for each input file.