Algorithm Design Laboratory with Applications

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Problem: *Deep Sea Research.*

You are a scuba diver collaborating to a deep sea research project. The supply of air in your tanks is limited and the researchers need at least W kilograms of some special kind of rocks.

You have a map listing the locations of the n rocks $r_1, \ldots, r_n$ of interest in the area. Each rock r_i has a weight of $w_i \in \mathbb{N}^+$ and needs $t_i \in \mathbb{N}^+$ minutes to be collected.

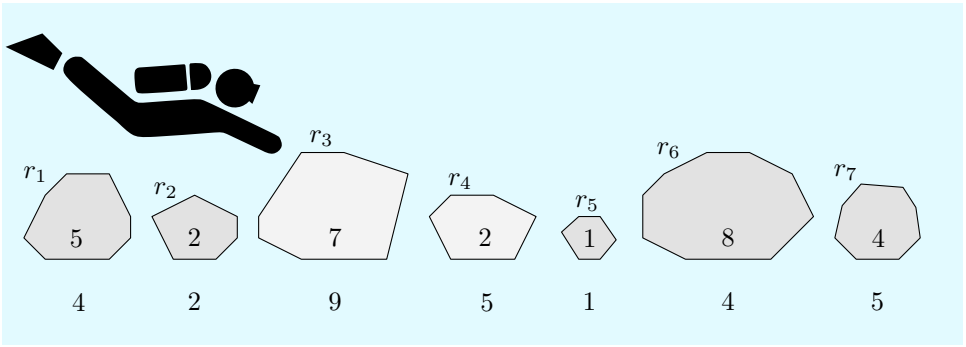
Design an algorithm that computes the minimum number M of minutes needed to collect a subset of rocks of total weight at least W.

Input. The input consists of a set of instances, or *test-cases*, of the previous problem. The first line contains the number T of test-cases. The first line of each test case contains the integers n and W. The next line contains the n integers $w_1, \ldots, w_n$. The third and fine line of the test case contains the n integers $t_1, \ldots, t_n$.

Output. The output consists of T lines. The i-th line is the answer to the i-th test-case and contains the integer M.

Assumptions. $1 \leq T \leq 10$; $1 \leq n \leq 2^{11}$; $1 \leq W \leq 2^{18}$; $\forall i = 1, \ldots, n,\ 1 \leq w_i \leq 2^8$; $\forall i = 1, \ldots, n,\ 1 \leq t_i \leq 2^{16}$.

Example.



Input (corresponding to the above picture):

```
1
7 20
5 2 7 2 1 8 4
4 2 9 5 1 4 5
```

Output (corresponding to subset of rocks $\{r_1, r_2, r_5, r_6, r_7\}$):

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16
```

Requirements. Your algorithm should require $O(nW)$ time (with reasonable hidden constants).

Notes. A reasonable implementation should not require more than 1 second for each input file.