

Algorithm Design Laboratory with Applications

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Problem: *Number station.*

A **number station** is a radio station that broadcasts sequences of numbers for espionage operations. You are a secret agent and you just received a transmission with the combination of a safe where vital intelligence is held. The transmission is encrypted and consists of a $n \times n$ matrix $A = (a_{i,j})_{i,j}$ of non-negative integers. As per your instructions, you are to recover the combination by counting the number of *non-empty contiguous submatrices* of A whose sum of elements is odd.

Time is key: design and implement an algorithm to *quickly* compute the safe combination.

Input. The input consists of a set of instances, or *test-cases*, of the previous problem. The first line of the input contains the number T of test-cases. The first line of each test-case is the integer n . The i -th of the following n lines describes the i -th row of A and contains the n integers $a_{i,1}, \dots, a_{i,n}$ separated by a space.

Output. The output consists of T lines, each containing a single integer. The i -th line is the answer to the i -th test-case and is the number of quadruples of integers (h, k, x, y) with $1 \leq h \leq k \leq n$ and $1 \leq x \leq y \leq n$ such that $\sum_{i=h}^k \sum_{j=x}^y a_{i,j}$ is odd.

Example. The safe combination associated with the matrix

$$A = \begin{bmatrix} 1 & 6 & 5 \\ 1 & 4 & 8 \\ 0 & 2 & 4 \end{bmatrix}$$

is 16, since A has 16 non-empty contiguous submatrices whose elements sum to an odd number.

Namely: $[1]$, $[5]$, $[1]$, $[1 \ 6]$, $[6 \ 5]$, $[1 \ 4]$, $[1 \ 4 \ 8]$, $\begin{bmatrix} 5 \\ 8 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 8 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 6 \ 5 \\ 4 \ 8 \end{bmatrix}$, $\begin{bmatrix} 1 \ 4 \\ 0 \ 2 \end{bmatrix}$,

$\begin{bmatrix} 1 \ 6 \ 5 \\ 1 \ 4 \ 8 \end{bmatrix}$, $\begin{bmatrix} 1 \ 4 \ 8 \\ 0 \ 2 \ 4 \end{bmatrix}$, $\begin{bmatrix} 6 \ 5 \\ 4 \ 8 \\ 2 \ 4 \end{bmatrix}$, and A itself. Notice that the submatrix $[1]$ is counted twice since it appears two times in A .

Input (corresponding to matrix A in the above example):

```
1
3
1 6 5
1 4 8
0 2 4
```

Output:

```
16
```

Assumptions. $1 \leq T \leq 10$; $1 \leq n \leq 512$; $\forall i, j = 1 \dots, n, 0 \leq a_{i,j} \leq 1024$.

Requirements. Your algorithm must have an asymptotic time complexity of $O(n^3)$.

Notes. A reasonable implementation should not require more than 1 second for each input file.